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Mechanical Evaluation of Symmetrical Extra-Oral Traction Appliances-an Experimental and Analytical Approach

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THE MECHANICAL EVALUATION OF SYMMETRICAL EXTRA-ORAL TRACTION APPLIANCES - AN EXPERIMENTAL AND ANALYTICAL APPROACH

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ABSTRACT

Laboratory and theoretical analyses were performed to determine the distribution of forces at the inner bow of a symmetric facebow. A constant facebow geometry was maintained for both analyses. A uniplanar displacement transducer system was employed to measure the force system produced at the inner bow attachment as a function of increasing load to the outer bows. Loading vectors formed a 9° angle to the midsagittal line. One series of investigations employed a rigid attachment while another modelled the clinical non-rigid circumstance with a headgear tube. The analytical model, based upon finite element theory, assumed a rigid attachment so as to permit comparison with the experimental results. Additional theoretical investigations included positioning the attachment point distal to the initial attachment position. The effect of bracket/attachment rotation was also studied by permitting attachment rotation through 1 degree (representing a mesial-out/distal-in tooth rotation).

Two material constants, the modulus of elasticity and the yield strength at .1% offset, were required inputs in the computer code of the theoretical analysis. Facebow wire specimens were tested in the as-received and the heat-treated ($850^{\circ}F$ for 15 minutes) conditions using an Instron Universal Testing Machine. Values for the heat-treatment wires were adopted for the theoretical analysis as all facebows tested were subjected to this treatment prior to experimental evaluation.

An elementary model, less complex in configuration yet constant

in material and cross-section, was evaluated experimentally and theoretically. A single centrally located point loading was used.

In all experimental facebow investigations distal forces were measured in addition to lateral forces tending to increase bucco-lingual arch dimension. Moments rotating the attachments mesial-out/distal-in were also recorded. Force system values for non-rigid attachment experiments were lower than the rigid attachment series, however the descrepancy was clinically insignificant. From the linear regression formulae derived, a load of 500 gms applied to each outer bow would produce a distal force of 479.9 gms, a lateral force of 95.3 gms, and moments between 1110.5 to 1047.2 gm-mm at the non-rigid attachment point.

The theoretical values obtained from the finite element analysis of the facebow were approximately 37-45% higher than the rigid attachment values produced experimentally. The descrepancy may be related to incomplete correction to the deflecting members of the measuring device, or inaccurate theoretical deflection response at the interface between the .045" wire with the inner arch tube.

The theoretical evaluation of moving the attachment point distally by 7 mm reduced the moments and lateral forces by approximately 30%. Attachment rotation dramatically affected the force system where .6 to .8 degrees rotation effectively negated the moments and reduced the lateral force by about 60%.

Force systems produced by the divergent arch were substantially higher than the facebow although the direction of the force components were the same. The theoretical values for the moments were about

25% higher than the experimental data, less descrepant than the values from the facebow investigations.

The broad anterior solder joint of the facebow contributes to a more rigid inner arch, and compared with the divergent arch a reduction in the lateral and angular deflection of the appliance is observed.

THE MECHANICAL EVALUATION OF SYMMETRICAL EXTRA-ORAL TRACTION APPLIANCES - AN EXPERIMENTAL AND ANALYTICAL APPROACH

INTRODUCTION

Statics is the study of a system of forces in equilibrium on a body at rest (or in uniform motion). Statics can be applied to orthodontic appliances producing forces upon the tooth and supporting structures. However, due to the large number of supports in the system, the force systems on many orthodontic appliances are statically indeterminant. Consequently, the force systems delivered by extra-oral traction devices are suited to laboratory investigation.

In the past studies of extra-oral traction appliances have primarily involved cephalometric or clinical evaluation with very little appreciation for the mechanics of the appliance. The literature is replete with empirical data on the subject while deficient in the qualitative and quantitative nature of the force systems produced by the facebow.

Extra-oral traction devices are used in the field of orthodontics primarily to maintain anchorage and to actively move teeth by guiding their pattern of eruption. It is imperative that we understand the physics of the extra-oral traction appliance so as to maximize its clinical manipulation and to predict the desirable and undesirable forces being produced.

It is the purpose of this study to determine the force systems produced by facebows under certain loading conditions by utilizing an analytical model as well as an experimental investigation.

LITERATURE REVIEW

One of the first references to the use of extra-oral traction appeared in 1841 by G. S. Gunnell. He claimed to have used it as far back as 1822 at the suggestion of Horace H. Hayden. It was first used as a remedy for the protrusion of the mandible, consisting of a small block of ivory tied to the lower teeth in addition to a cap and straps fit to the back of the head. The ivory block was placed between the posterior teeth, tied to the lower arch and the cap and straps drawn as tight as the patient could bear. The appliance was designed to "press the joint ends of the lower jaw backwards and downwards and press the chin backwards and upwards, the block of ivory acting as a fulcrum". He claimed to have restored the face and jaws to their proper symmetry in one week, although occasionally it would take three to six weeks or even longer.

In 1836, a German by the name of J. F. C. Kniesel (the inventor of the modern impression tray) was the first to publish the idea of occipital anchorage, where the back of the head was employed for anchorage.

The earliest reference to extra-oral traction for distal movement of teeth appeared during a discussion on the treatment of dental irregularieties at the American Dental Convention in 1863. Dr. Searle described a case that occurred in the practice of Dr. Christopher Brewster, an American dentist who practiced in France. Apparently a Russian nobleman, plagued with a disturbing anterior dental deformity, was seeking treatment from physicians and dentists but to no avail. He approached Brewster who agreed to treat the problem if his instructions were obeyed. The appliance consisted of a "saddle upon the back of the head with a ligature passing directly over the front teeth". Within an eight month period so great a change had taken place that the Russian nobleman, upon returning to his home, was unrecognized by his friends.

At a meeting of the Society of Dental Surgeons of the city of New York in 1867, Dr. Kingsley, while talking on the subject of dental irregularities remarked about the difficulty of treating "overhanging superior incisors". He described one such case and attempted to retract and intrude the incisors with an apparatus consisting of a gold plate resting against the incisal edges of the teeth. Elastic straps were attached to the gold plate and the ends of the straps were fastened to a cloth cap resting upon the head.

At a New York Odontological Society meeting in 1881, S. G. Perry described an apparatus designed to retract the incisors and four bicuspids en masse into the extraction sires of the upper first molars. The case was fitted with a vulcanite plate covering the front teeth. Two steel wires attached to the plate projected out at the corners of the mouth and elastics were fastened to the wires. The elastics inturn were fastened to silk bands which were carried around the head, one above and one below the ears. He reported that very rapid progress was made in carrying back the incisors and four bicuspids, attributable to achieving firm occipital support.

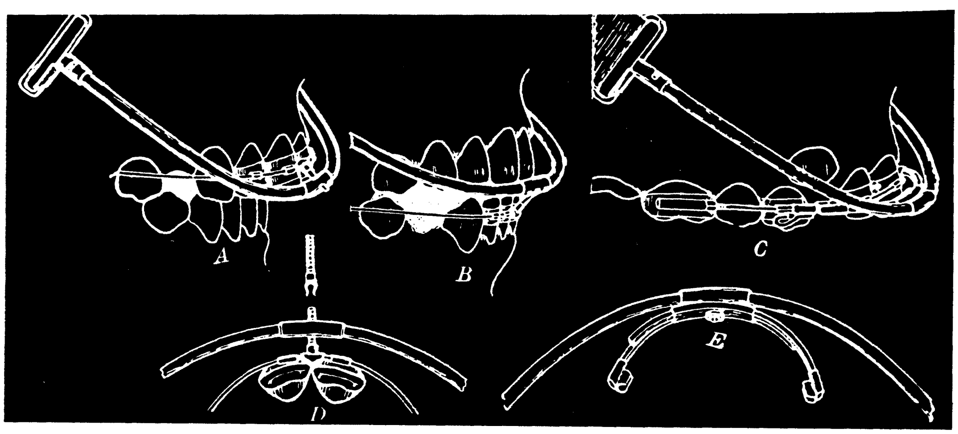
In 1904, Calvin S. Case published a paper wherein he described in detail the application of occipital forces. In doing so he contributed some of the earliest recorded information on the occipital headcap, in addition providing numerous illustrations of the headcap apparatus with various modifications. Case also laid down the requirements of the headgear. He stated that "a headgear apparatus:

1/ should be worn with the least possible production of discomfort.

2/ should consist in possibilities of adjustment, so that it can be perfectly fitted to the individual case.

3/ should lie smoothly on the surfaces of the head and face, with no projecting portions or prominences to produce uneven and annoying painful pressures while the head is at rest upon the pillow.

4/ should enable the patient or attendant to gradually increase or diminish the force."



Case's headpiece consisted of thin metallic ribbons with sliding gear so that it could be fitted to the head. His inner and outer bows came in various forms. Figure A and D illustrate the assembly he utilized for maxillary anterior retraction consisting of a threaded post-rest allowing comfortable positioning for the lips. Figure B shows a similar apparatus used for retracting the lower anterior segment. Figure C and E illustrate a headgear employed for retruding any or all the buccal teeth. Case's works marked an important event in orthodontia as this was probably the first successful distal movement of the molars.

The outer bow and inner bow of his facebow were attached and an adjusting screw at the junction provided adjustment of the length of the inner bow for molar or bicuspid attachment. It appears that Case advocated the use of this type of headgear for the correction of posterior teeth that drifted mesially subsequent to premature extraction of deciduous teeth. Case's occipital headgear was indicated in cases where anchorage was critical, in cases requiring distal movement of molars, and in cases requiring retention. He, at this time, realized some of the limitations of the occipital headgear when he stated "it is not advisable, however, after the eruption of the second molars to attempt an extensive distal movement of back teeth that have not been moved forward by natural or artificial forces".

In the same article of 1904, Case also presents what interestingly is probably the first attempt at applying a unilateral force with occipital headgear. He states:

> "In an apparatus now worn by the wife of a Chicago dentist all of the occipital force is successfully directed to the distal movement of a single upper right molar. This is accomplished with the pivital point properly adjusted to one side of the center of the headgear bow".

This description would seem to closely resemble the present day swivel offset unilateral facebow. The basic design of his occipital facebow closely resembles todays commercially available symmetrical units.

Prior to publishing his works on headgear in 1904, Case had developed another means of "anchorage" known as the "mesiodistal intermaxillary anchorage", a principle presented in February of 1893. Intermaxillary anchorage entailed the use of elastics to provide intermaxillary reciprocating forces. In Class II malocclusions distal forces to maxillary anteriors and mesial forces to mandibular posteriors were applied. Class II malocclusions involved applying mesial forces to maxillary posteriors and distal forces to mandibular anteriors. It has been erroneously thought that Henry A. Baker originated the principle (hence "Bakers anchorage"), this oversight attributable to Edward H. Angle's reference to Baker as the founder of the concept instead of Case. It is important to note that Case advocated that intermaxillary elastics be employed as an auxillary to the occipital headgear, rarely were intermaxillary forces indicated exclusively. Case also recognized the limitations of intermaxillary elastics when he spoke of the extruding action as one of the main objections of intermaxillary force application.

Unfortunately, the profession did not attend to Case's teachings and as the popularity of Edward Angle increased, intermaxillary forces became the primary means of attaining anchorage. Consequently, occipital headgear was ignored and succumbed to the popularity of intermaxillary anchorage, the value of which was grossly overestimated.

So it was that prior to the inception of mesio-distal intermaxillary anchorage in 1893, some form of extra-oral occipital traction had been the primary means of moving teeth distally or maintaining anchorage. The apparatus received no further development during the early part of the twentieth century and it was generally discarded for a period that lasted approximately forty years.

However, in 1934, Oppenheim in Vienna resurrected the headgear quite by chance. A certain actress with protruding anterior teeth required orthodontic correction but the treatment could not interfere with her professional obligations. Oppenheim suggested the use of the headcap and she obliged. After several office visits and much complaining of soreness, chewing problems and sleeplessness from pain, the patient finally became delinquent in coming back to see him. Several months later, however, she returned after performing on tour in Europe and she presented with an improved facial appearance and an end to end molar relationship which formerly had been a Class II relationship. Apparently the teeth had ceased to be sore, the patient wearing the headcap during her months of absence. Oppenheim's apparatus consisted of two molar bands on the maxillary first molars and a rigid high labial arch attached by elastics to an occipiral headcap. He attributed the success of distal movement of the molars to the application of light intermittent force which his research findings indicated to be the most effective means of moving teeth from a biological standpoint.

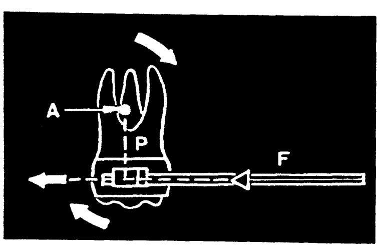
In 1938, Brodie, et. al., published a preliminary report on their first cephalometric studies of orthodontic results noting changes in the occlusal plane when intermaxillary elastics were used in the treatment of Class II and III malocclusions. The occlusal plane in almost all cases studied returned to its original position after orthodontic treatment had ceased. In addition, the mesial movement of the mandibular teeth, in response to the intermaxillary forces, tended to create bimaxillary protrusions and disturbed axial inclinations of teeth which in most instances relapsed subsequent to treatment.

With research beginning to indicate the ineffectiveness of intermaxillary anchorage, Oppenheim's headcap treatment began to generate some interest in research circles. However, it wasn't until 1945, when Oppenheim came to the United States, that his headgear began to stimulate serious research in this area. By 1946, Walter N. Epstein, a student of Brodie's in Illinois published a cephalometric study of molar relationship changes following only headcap treatment in Class II malocclusions. He concluded that extra-oral traction approached an ideal in treating these malocclusions because the side effects such as mesial tipping of mandibular teeth and canting of the occlusal plane were avoided. In addition he stated that the correct relationship between maxillary and mandibular molars was obtained by holding maxillary molars stationary (or by distal movement of molars) in the forward growing maxilla while permitting normal growth and development of the mandible. Shortly thereafter the revival of extra-oral traction gained in momentum.

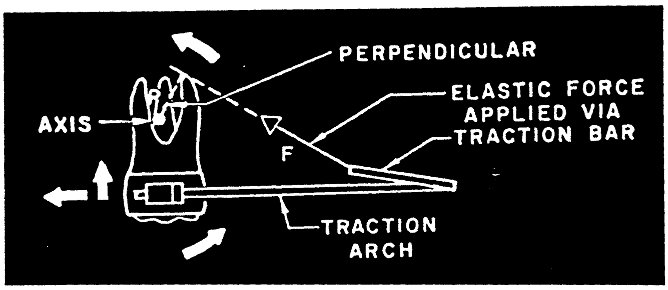
In 1947, Silas J. Kloehn published a paper discussing his clinical findings on extra-oral traction utilization. In this paper he also described the technique of fabricating the facebow out of .045" round stainless steel wire, the basic design of which is very similar to the modern "Kloehn facebow". He concluded that extra-oral traction was instrumental in guiding alveolar growth and tooth eruption to obtain a better facial balance, that it reduced the severity of the malocclusion and also reduced the treatment time.

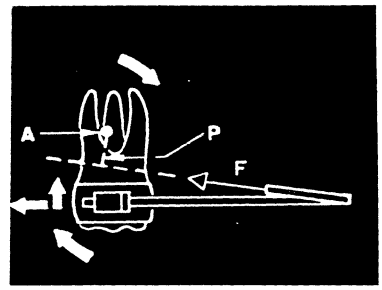
In an article published in 1953 Kloehn again presented more clinical evidence for utilizing extra-oral traction but in this instance he advocated using a cervical strap instead of an occipital headcap. He also presented two adjustments that could be made to the facebow to achieve different molar axial inclinations. If the outer facebow were to lie above the inner arch wire then distal root movement would result. If, on the other hand, the outer bow was placed below the inner archwire, distal crown movement would be expected. This was probably the first instance in the literature where some attempt was made to understand the basic mechanics of the facebow. Soon after this article was published extra-oral traction became widely accepted amongst orthodontic practitioners.

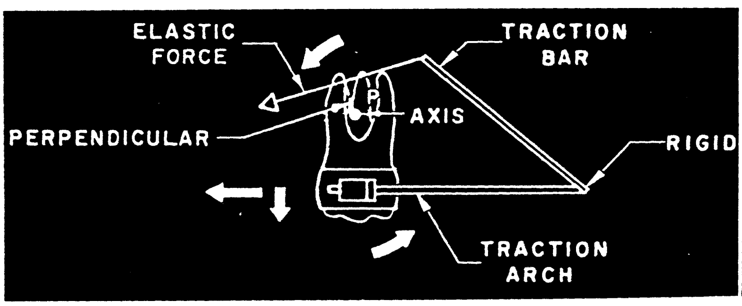
Gould, in 1957, was one of the first to study the facebow bio-mechanically and he examined it primarily in the sagittal plane of space. He assigned an axis to the molar teeth about which rotation would occur given the properly applied force system. Gould deduced the direction of the elastic force and its relationship to the axis ("of rotation") is what ultimately determined the type of tooth movement. If the line of force was parallel to the occlusal plane then no extrusion or intrusion would occur because no vertical component existed.

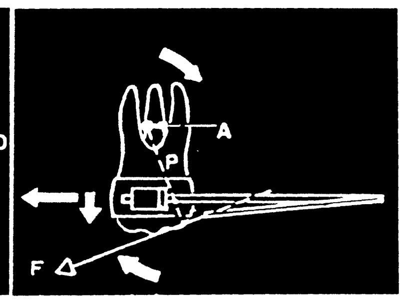


In a situation where the line of force was above, below or through the axis, but not parallel to the occlusal plane, then a vertical component would be introduced. An intrusive force would exist if the line of force were to pass superior to the OP. An extrusive force would exist if the line of force were to pass inferior to the OP.



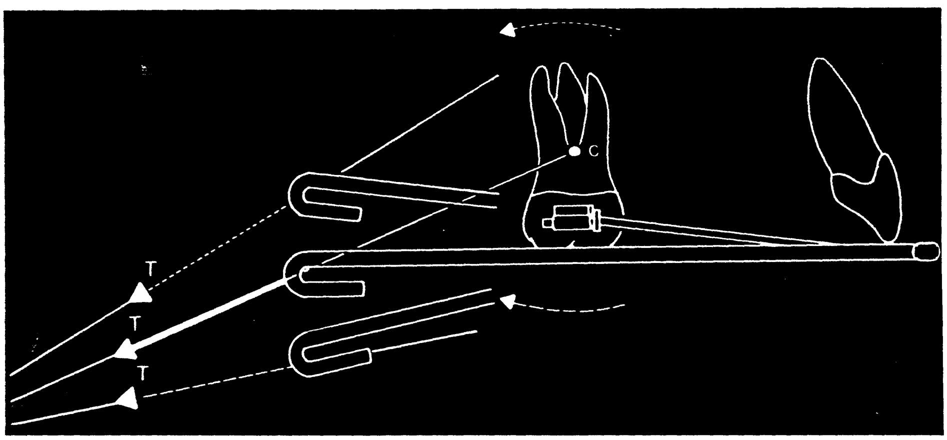


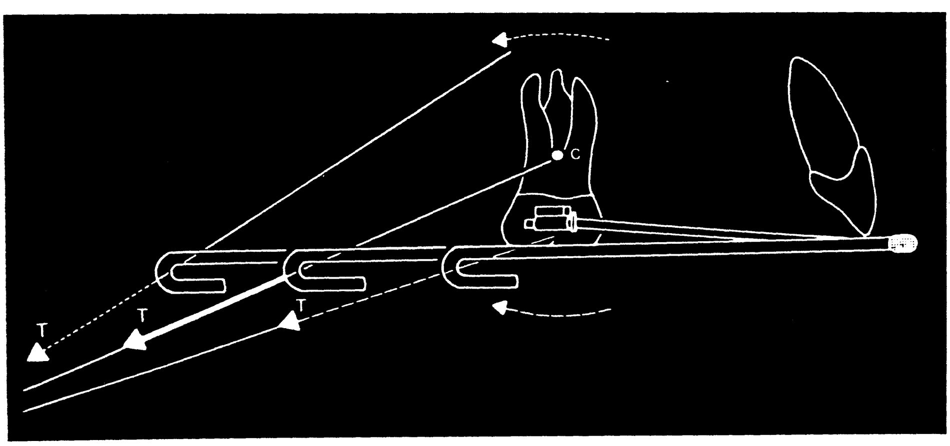




A couple or moment is produced when the line of force passes above or below but not through the axis of the tooth while a force above the axis tends to produce a distal root and a mesial crown movement in addition to the distal displacement. A force passing below the axis produces mesial root and distal crown tip in addition to the distal movement.

The magnitude of the moment is dependent upon the force and the perpendicular distance from the line of force to the axis of the tooth. When the line of force passes through the axis no moment is introduced and no rotation of the tooth occurs. If the line of force passing through the axis is parallel to the occlusal plane a pure distal translatory displacement results. However, any line of force passing through the axis that is not parallel with the occlusal plane introduces a vertical component. Gould also indicated how alterations to the outer bow length and changes in angulation between inner and outer bows can change the line of action of the force.





Since Gould, numerous authors have expanded upon the basic biomechanical principles. One significant alteration to the model has been the replacement of the axis of rotation with two centers known as the center of rotation and center of resistance.

According to Burstone, the center of resistance is a fixed point on a tooth, unchangeable by external force application, through which the resultant of constraining forces act upon. The center of rotation is that point about which the tooth will rotate and, unlike the center of resistance, can exist anywhere between the center of resistance and infinity depending upon the external forces and moments.

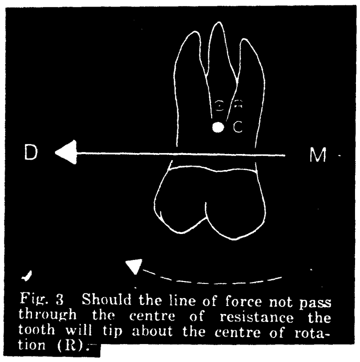
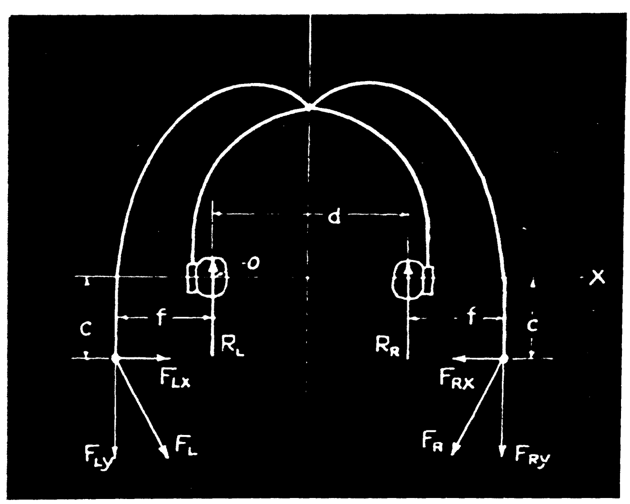


Fig. 3 Should the line of force not pass through the centre of resistance the tooth will tip about the centre of rotation (R).

When a force is applied to a tooth and its line of action doesn't pass through the center of resitance, then the tooth will rotate about the center of rotation. The centers of resistance and rotation coincide when the line of action of the force passes through the center of resistance, no rotation occuring.

Another first with regard to the biomechanical approach to understanding headgear appeared in 1958 when Haack and Weinstein discussed the force distribution of centric and eccentric facebows in the horizontal plane. They stated that if Force$_L$ and Force$_R$ are equal the resultant force R, that is the force that would completely replace Force$_L$ and Force$_R$, would be on the midsagittal midline and in the same direction with a total magnitude of Force$_L$ and Force$_R$. However, it is the direction of Force$_L$

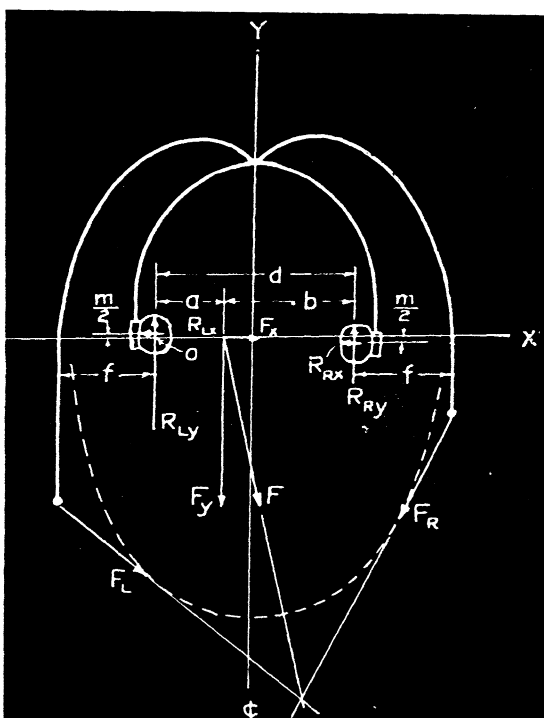


and $Force_R$ that is of utmost importance because the geometry formed by the elastic bands determines the resultant. The relationship of the resultant to the midsagittal line determines the reactive forces at the right and left molars.

They also maintained that the conditions of equilibrium must be satisfied before one could understand the action of the facebow forces on the teeth. To achieve equilibrium in one plane of space:

1. $F_x=0$ The sum of the forces in the x direction is zero.
2. $F_y=0$ The sum of the forces in the y direction is zero.
3. $M_o=0$ The sum of the moments about any point is zero.

Where one facebow arm is longer than the other arm, one applies equal forces yet they are not symmetrical in direction. This produces a resultant that is not along the midsagittal line and consequently the reactionary forces on the right and left molars are different.



The equilibrium equations are applied as follows:

"1: $F_X=0$

$F_X - R_{RY} = 0$

Positive signs are for forces acting upward and to the right, and negative signs are for forces acting downward and to the left. R_{RX} can be assumed to equal R_{LX}. This assumption can be explained by the use calculus and the methods of elastic energy. This equation may now be rewritten:

2. $F_Y=0$

$-F_Y + R_{RY} + R_{RY} = 0$

3. $M_o=0$

A moment can be taken around any point, and simplification suggests the point o on the X axis at the left molar. Therefore:

$$-F_Y \times a - R_{RX} \times m/2 + R_{RY} \times d + R_{LX} \times m/2 = 0$$

Since $R_{RX} = R_{LX}$, this equation reduces to $-F_Y \times a + R_{RY} \times d = 0$.
Solving for R_{RY}, $R_{RY} = \frac{F_Y \times a}{2}$. Substituting in equation No. 2

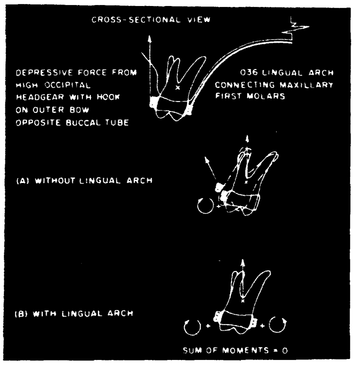
$$-F_Y + R_{LY} + \frac{F_Y \times a}{d} = 0$$

Solving for R_{LY}, $R_{LY} = F_Y - \frac{F_Y \times a}{d} = \frac{F_Y \times b}{d}$.

Now comparing these two forces, R_{RY} and R_{LY}, it is seen that the force on the left molar is of greater magnitude than the one on the right, because distance b is greater than distance a. The ratio of these forces would be the ratio of the distances a to b".

It should be noted that a net lateral force is introduced in this system, although the force is small. This lateral force increases as a function of the cosine of the angle that the resultant forms when it intersects the midsagittal line. In addition to discussing mechanics, Haack and Weinstein emphasized one fundamental principle concerning a rigid body. They stated that "the internal configuration of a rigid body (and they assumed the facebow was a rigid body) does not affect the distribution of the external forces on the body". Applying this principle they were able to show that the soldered offset unilateral facebow could not develope an asymmetric force system. Wherever the rigid attachment between inner and outer bows is made the reactionary forces on both the left and right molars will be equal if the loading forces are equal in direction and magnitude.

Kuhn, in 1968, briefly discussed the resolution of headgear forces and suggested the use of a lingual arch between maxillary molars whenever an occipital headgear with a large intrusive component was utilized. The intrusive component acting upon the buccal tube of the molar bracket would tend to "roll out" the crown to the buccal because the force is placed lateral to the center of resistance (in the coronal plane).



In the absence of the lingual holding arch the lingual cusp of the molar would tend to roll out and make premature contact with the inclines of the mandibular buccal cusps, thus rotating the mandible open. Oosthuizen, et. al., in 1973, applied simple algebra to resolve headgear forces, as seen in the sagittal plane, into horizontal and vertical components. With analysis similar to Haack and Weinstein they were able to quantify the intrusive/extrusive and distal forces, in addition to the moments about the center of resistance.

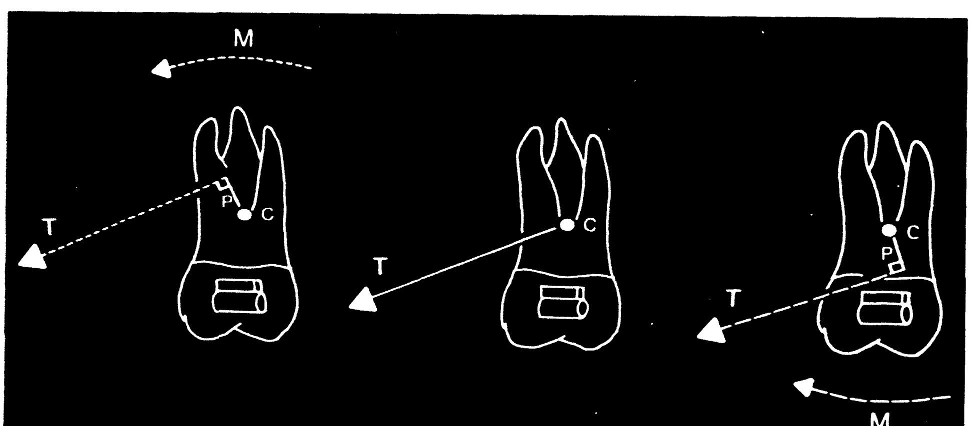
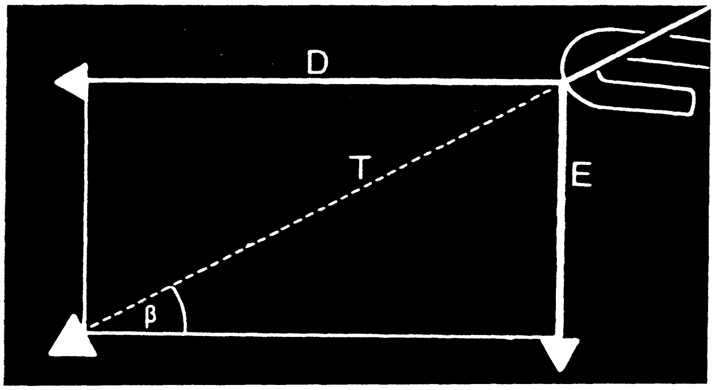


Fig. 8 Analysis of the tipping forces.

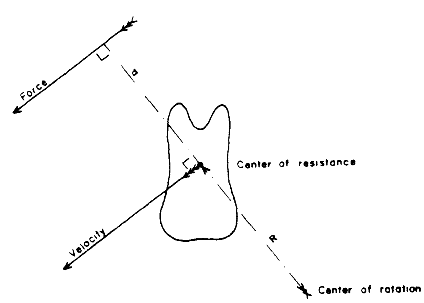
M = T x P Moment = Force x Perpindicular distance from the center of resistance to the line of action



sin B = E/T
cos B = D/T
tan B = E/D

Thus E = T sin B
Thus D = T cos B

Worms, et. al., in 1973 expanded our understanding of the centers of rotation and their importance in extra-oral force delivery. Problems arose when they attempted to quantify molar movements subsequent to facebow therapy and they reasoned that this was related to varying centers of rotation of the molars. When a single force is applied to a tooth (not through the center of resistance and without a couple) an instantaneous center of rotation is created somewhere between the center of resistance and infinity. The shortest perpendicular distance between the force vector and the center of resistance determines the center of rotation. The instantaneous center of rotation will lie on a line perpendicular to the force vector. Mathematically they went on to show that the center of rotation is inversely related to distance and independent of force. They stated:



"F = Force, V = Velocity, X = Angular Speed, Y = Viscosity Coefficient

Z = Viscosity, R = Center of Rotation, D = Distance

$R = V/X \quad V = F/Y \quad X = \frac{F \times D}{Z}$ Substituting $R = \frac{F/Y}{F \times D/Z} = \frac{Z}{D \times Y}$

The ratio of Z to Y is probably constant; therefore the center of rotation (R) is inversely related to distance (D) and independent of force (F).

Therefore, $R = \frac{\text{Constant}}{D}$ " .

They argued that headgears usually have constant sources of force direction but continuously changing force to center of resistance distances, therefore yielding instantaneous centers. In order for there to be a constant center of rotation the force vector would have to constantly change with the movement of the tooth and this, they felt, does not occur.

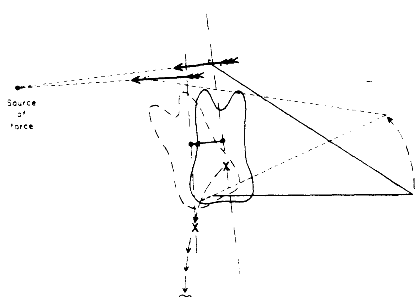
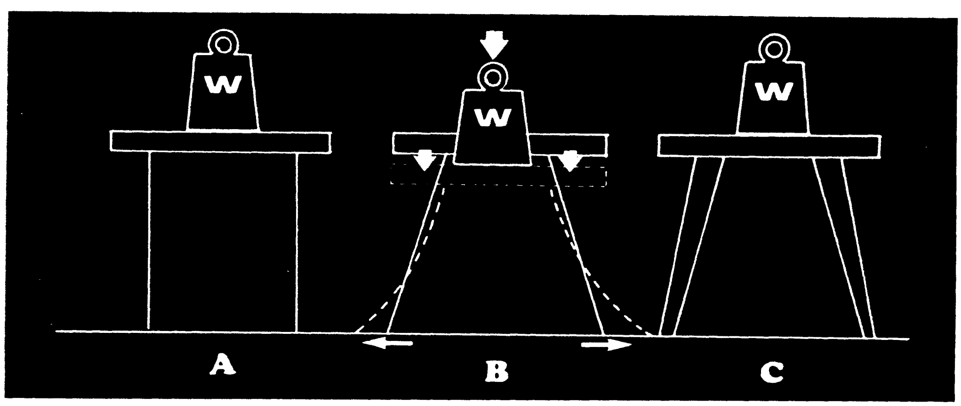


Fig. 7 Illustrates changing centers of rotation with a constant source of force. As the molar rotates, the facebow rotates. The perpendicular distance between the headgear force vector and the molar center of resistance decreases causing a migration of the center of rotation toward infinity.

Thus the center of rotation of the molar is extremely sensitive to the location and direction of the headgear force. When the force was applied through the center of resistance the tooth would translate because of an instantaneous center of rotation at infinity. As the perpendicular distance between the center of resistance and the force increases, the instantaneous center of rotation migrated toward the center of resistance. They stated "as the perpendicular distance of the force to center of resistance passed through the apices, the instantaneous center of rotation was at the occlusal surface of the crown. By doubling the above perpendicular distance of the force from the center of resistance, the center of rotation approached very near the center of resistance and consequently the apex and crown moved in opposite directions". They also noted clinically that erupted second molars in contact with first molars created a resistance to posterior movement and in effect altered the center of resistance. The force vector that caused translation in these instances tended to be closer to the crown rather than at the trifurcation indicating that the center of resistance was closer to the crown.

In 1979, Jacobson discussed extra-oral forces and touched upon the expansion force of the facebow's inner bow. He reasoned that if lightweight legs supporting a heavy body were parallel, no lateral forces or divergence of the legs would occur. If however, the legs were non-parallel and weight was placed on them, a "splaying" effect could be seen and a lateral force would result.



Utilizing a simple pulley system Jacobson attempted to examine lateral forces when weights representing distal forces were placed on the inner bows. Although no values were quoted he reported that a .045" inner bow diameter facebow with divergent legs showed considerable expansion, moreso than legs that were parallel. He also stated that facebows with stiffened or reinforced anterior sections (at the union of outer and inner bows), or facebows constructed of heavier wire exhibited little facebow expansion even with the application of heavier forces up to three pounds per side.

Lateral forces, created by the inner bow's tendency to diverge when activated, were discussed briefly by Anderson in 1978 and in more detail by Houghton in 1979. Utilizing a strain gauge transducer system developed in 1973 at the University of Connecticut Health Center, Houghton analyzed unilateral facebows and noted the "archial expansion" effect of the inner bows. The expansion, he noted, was also occuring in the non-unilateral facebows such as the Kloehn bilateral facebow. With an activation of 250 gms on each outer bow a facially directed lateral force of 24 gms was observed. He noted facebows with a broad solder joint between inner and outer bows were characterized by a more rigid inner bow which minimized the archial expansion. The narrow solder union allowed for a more flexible inner bow and the lateral forces generated were significantly higher.

In 1975, Terlingen of The Netherlands, evaluated the force systems of facebows with force/couple strain gauges; he too observed that lateral forces and moments were generated by expansion of the inner bow when the facebow was loaded.

Haack and Weinstein in 1981 studied the bucco-lingual forces produced by extra-oral appliances using an experimental and analytical approach. The bilateral moments were also examined in the theoretical analysis. Conditions tested included the effect of a loose fit and precise fit between arch and tube, the effect of different molar stop lengths, and the effect of combinations of inner bow radius, length and angle of convergence. Buccal lateral forces were found to be higher if the legs of the inner bow were more divergent, but lower as the molar offset was increased. A loose fit between arch and tube produced a lower lateral force when compared to a precise fit relationship. Theoretically, no moments were generated in the loose fit arrangement but were produced in the precise fit relationship. They reported a high degree of correlation between the experimental and theoretical values for lateral forces.

SUMMARY

Although experimental investigation and clinical observation since the mid-ninteenth century has broadened our knowledge of the application of the facebow, none of the experimental studies have comprehensively examined the mechanics of the facebow in all planes of space, nor have they employed a sound mathematical model for comparison (the exception being the recent mathematical model proposed by Haack and Weinstein). The use of accurate instrumentation for force system evaluation and the application of an analytical model based on engineering principles may produce an appreciation for the mechanical nature of the facebow.

OBJECTIVES

The objective of this investigation was to evaluate the mechanical behavior of the facebow and determine the distribution of forces at the inner bow terminals during symmetrical loading, paying particular attention to the lateral forces and the moments generated. A constant facebow geometry was maintained throughout the experimental and theoretical analyses. The specific objectives of the project are listed:

A. Perform a laboratory analysis of the symmetrical facebow to:

1. evaluate the magnitude of the bilateral distal forces at the inner bow terminals.
2. determine the relationship of the forces and moments produced at the inner bow terminals as a function of outer bow load.
3. evaluate and compare force systems from rigid attachment and non-rigid attachment of inner bow terminals.

B. Perform a theoretical analysis of the symmetrical facebow to:

1. determine the magnitude of the forces and moments at the inner bow terminals.
2. determine the relationship of the forces and moments generated at the inner bow terminals as a function of load.

C. Compare the results for rigid attachment from the experimental and the theoretical analyses.

HYPOTHESIS

For any given distal load to the outer bows of a symmetrical facebow a distal force, a lateral force, and a moment will be produced at each inner bow attachment.

MATERIALS AND METHODS

I. EXPERIMENTAL

A. Uniplanar Displacement Transducer System

An apparatus for measuring uniplanar force systems produced by orthodontic appliances has been constructed at the University of Connecticut, Department of Orthodontics (Figure 1). The device has the capacity to measure portions of the force systems produced by an orthodontic appliance secured at two points of attachment. Vertical force (F_y) is measured at one attachment point. Horizontal force (F_x) is measured at the second. Couples within the plane defined by the above mentioned forces are measured at both points of attachment (Figure 2). All measurements are independent of one another.

The linear load-deflection characteristic of a cantilever beam is utilized to measure force (Figure 3). The position of the cantilever beam free-end is transduced to a voltage by a linear voltage displacement transducer (LVDT). A load applied to the attachment point deflects the cantilever beam, resulting in a second voltage recording. Calibration is accomplished by evaluating the relation between change in voltage and applied load.

An angular displacement transducer (ADT) in series with a torsional beam is employed to measure a couple. The ADT outputs a voltage corresponding to the angular position of the attachment point. A couple acting about the attachment point causes a rotation of the torsional beam, resulting in a second output voltage. The relation between the couple magnitude and output voltage difference is the functioning calibration curve for the transducer-torsional beam system.

Under software command each transducer output voltage is processed by an analog to digital converter, and recorded by a Computer Automation Alpha-16 minicomputer. A force system measurement consists of two samplings of all four transducers; one prior to loading, the second after loading. The difference between the two readings, when operated on by the corresponding calibration relation yields a component of the applied force system.

B. Attachments

The experimental runs consisted of two series of tests. The first series utilized a clamp receptacle permitting rigid fixation of the inner bow terminals to the measuring device. The second series of experiments employed a commercially available headgear tube. The tube welded to a steel rod permitted a certain degree of movement at the interface between the inner bow terminal and the headgear tube, thereby modelling the "non-rigid" clinical circumstance. Figure 4 illustrates the two attachments.

C. Loading Device

To approximate the clinical force vector geometry a loading device was necessary. An aluminum structure was constructed to allow modelling of the relationship of the force vectors to the back of the head/neck. The design permitted adjustment of two ball bearing pulleys for loading and calibration purposes. The loading device was oriented to allow loading in the plane of the facebow, and firmly anchored.

D. Facebow Specifications

For each trial one Kloehn facebow (Unitek #320-451) was used.

*Ormco # 182-4522 Upper Molar Bracket-Tube Combination

The facebow consisted of a .045" diameter inner bow wire and a .072" diameter outer bow wire soldered to a .045" inner diameter inner arch tube press fit to the .045" wire. The facebow was configured to approximate conditions typically seen in the clinical setting. The .045" wire and tube were composed of 304L steel, a low-carbon steel. The .072" wire was 455 steel. Facebow dimensions and geometry remained without change throughout the experiments. The loading conditions and the type of attachment were varied.

The distance from the anterior most part of the inner bow to a line connecting the molar offsets (molar stops) was 36 mm. The distance between the molar offsets was 52 mm. The first bend of the molar offset was a 135^{o} bend (relative to the mid-sagittal line) made 3 mm before the second bend. The second bend, the point of attachment to the measuring device receptacles, was made 7^{o} to the mid-sagittal line (in accordance with ideal arch principles). Hooks for attaching the loads were contoured into the outer bows at a point 45 mm from the most anterior part of the inner bow. The outer bows were bent symmetrically to lie 70 mm away from the mid-sagittal line. Figure 5 illustrates the geometry of the configured facebow.

E. Loading Conditions and Trials

The force loading vectors fromed a 9^{o} angle to the mid-sagittal line. Dead weight loadings from 100 to 500 gms in increments of 100 gms were applied. In the rigid attachment series a monofilament line was used to connect the weights to the hooks of the outer bow. Due to its negligible elongation under load .010" stainless steel wire was employed to connect the weights to the outer bows in the non-rigid attachment experiments. This series utilized an elevating platform permitting synchronous loading of both outer bows with the movement of the platform (Figure 6). To maintain a more constant contact point in the "unloaded state" a 50 gm pre-load was used.

Two facebows were constructed for each series of attachments. For the rigid attachment series five readings were taken for each loading condition. With the non-rigid attachment ten readings were recorded for each loading. Corrections were made to the raw data for deflections inherent to the measuring device.

F. Correction for Receptacle Displacement

The cantilever beams and torsional shafts of the uniplanar displacement transducer system displace during loading. Consequently, geometry changes at the attachment receptacles alter the facebow geometry. Due to the large cross-section of the wires small alterations in the geometry result in appreciable errors in the force system magnitude.

Six possible displacements were observed:

1. Right Transducer - Horizontal
   - Vertical
   - Angular
2. Left Transducer - Horizontal
   - Vertical
   - Angular

Each facebow was tested to correct for the deflections of the measuring instrument. The attachment receptacles were displaced with the facebow in position. Displacement was achieved by loading one receptacle in one direction through its center of rotation. For each load a recording was made of the change in output of each transducer. Independent vertical, horizontal and angular displacements were imparted to each receptacle. Plots of force system vs load were constructed. Corrections for the six displacements were applied to the raw data of each sample.

G. Material Properties of the Facebow

Certain properties of the facebow wire, specifically the modulus of elasticity and the yield strength, were required inputs for the theoretical analysis. The orthodontic literature cites substantial differences between the commercially quoted material constants, and the experimentally determined material constants for orthodontic wires[31]. To alleviate this possible inaccuracy the modulus of elasticity and the yield strengths at .1% and .2% offset were measured using an Instron Universal Testing Instrument.

1. Instron Testing Machine

The Instron Universal Testing Machine is an automated device for testing wire specimens in tension or compression (Figure 7). Axial load on the test sample is transduced by a precalibrated strain gauge network (load cell). Change in length of the test sample is transduced by an extensometer secured to the sample. A plot of axial load vs length change is recorded as the sample is continuously strained by the motion of a crosshead securely fastened to the sample.

2. Experimental Design

The facebow wire specifications were tested in two conditions:

a. As-received
b. Heat treated

To reduce the variation due to the history of cold working and drawing, the wires were heat treated. It is hypothesized that heat treating reduces the residual stress within the wire thereby standardizing the samples[32]. The heat treated samples were subjected to a heat treatment of 850^{o}F for a period of 15 minutes. Some samples were subjected to a recrystallization heat treatment of 1850^{o}F for a period of 5 minutes. This heat treatment was kept below the transition temperature of the material thereby avoiding a phase change. Two lots of facebows were tested. The number of samples tested was dependent upon availability and ability to test without slippage. A sample was not re-tested if slippage occurred. Figure 8 illustrates a wire sample secured to the grips and the extensometer that measures the change in length of the sample.

A 200 kg load cell was used for samples of .045" diameter. A 500 kg load cell was employed for the .072" diameter samples. A cross-head speed of .05 cm/minute and strain magnification of 1000:1 was common to all tests.

The modulus of elasticity and yield strengths at .1% and .2% offset were derived from the Instron chart output. The plot slope, divided by the nominal cross-section and multiplied by the gauge length is the sample modulus. The load at the respective offset divided by the original cross-section is the yield strength. Test results were employed as inputs to the theoretical analysis.

II. THEORETICAL

A. Finite Element Analysis

A computer code for determining force systems delivered by orthodontic appliances has been constructed by Dr. Herbert Koenig at the University of Connecticut, Department of Engineering. The analytical model is based on finite element theory.

The appliance is modelled as a series of linear beam elements. Curvature is allowed at the elemental endpoints to maintain continuity. The equilibrium relations including any surface tractions are written. The deformation equations and load-deformation equations of each element are expressed in finite difference form. The load-displacement characteristics of each element is a matrix composed of the above relations.

The relationships between loads and displacements of one element to adjacent elements are established with the conditions of equilibrium and continuity at the interfaces. Simultaneous solution of these relations with any constraining boundary conditions yields the loads and displacements of the appliance.

B. <u>Facebow Coordinate System</u>

The geometry of the facebow used in the experimental series was employed in the finite element analysis. To obtain the coordinates of this geometry a facebow was configured and painted with 98 points along its surface. The facebow was photographed against a millimeter grid and the negative enlarged 2X. The two dimensional cartesian coordinates were determined by the position of the points along the grid. These coordinates were input to the computer; lines connecting these points became the elements of the theoretical model. Figure 9 illustrates the geometry of the undeformed facebow as reproduced by the computer.

C. <u>Attachment and Loading Conditions</u>

A rigid attachment of the inner bow was assumed for the analysis.

One coordinate was chosen on each arm of the inner bow as the point where the force system would be determined, this coordinate approximating the point of measurement in the experimental series. Loading conditions equivalent to the experimental conditions were maintained.

To determine the effect of bracket/attachment rotation on the force system, the point of attachment was allowed to rotate through 1 degree. This effect may represent play between the wire tube interface and/or a small amount of tooth rotation. Rotation was permitted at both attachments and the direction models the clinical setting of a mesial-out/distal-in tooth rotation. A load of 500 gms was applied.

The effect of positioning the attachment point was also studied. The force system was evaluated with the attachment point 7 mm distal to the molar stop. A load of 500 gms was employed.

III. EXPERIMENTAL - THEORETICAL CORRELATIONS

A. Rationale

To test the equipment and methods a more elementary model was chosen. This allowed a comparison to be made with the theoretical model using an appliance less complex in configuration yet constant in material and cross-section.

B. Experimental

The uniplanar displacement transducer system was employed for this series. A divergent arch, with dimensions similar to the inner bow of the facebow, was configured without the molar offsets. The rigid attachment receptacle was used. The divergent archwire (.045" diameter) was configured and subjected to a heat treatment of 850^{o}F for a period of 15 minutes. A centrally located single point loading was used. Loads were varied from 100 to 1000 gms in 100 gm increments. Samples were taken with the divergent arch fixed rigidly to the receptacles and then the same procedure was repeated with the arch inverted, the same loading point being maintained (Figure 10).

C. Theoretical

Coordinates for the divergent arch were assigned as with the facebow. A rigid attachment allowing no rotation or displacement was assumed. The force system was determined at one point on each arm of the arch. Loads were applied to a single centrally located point, consistent with the experimental loading. The experimentally determined values for the modulus of elasticity and the yield strength were used. Figure 11 illustrates the geometry of the undeformed divergent arch as reproduced by the computer.

RESULTS

I. EXPERIMENTAL

A. Facebow

Tables I and II of Appendix I show the uncorrected force system values for Facebows I and II tested in the rigid attachment series of laboratory investigations. Tables III and IV of Appendix I display the uncorrected force system values for Facebows I and II run in the non-rigid attachment series of experiments. In all investigations distal forces were recorded in addition to lateral forces tending to increase the bucco-lingual arch dimension. All moments measured tended to rotate the attachment mesial-out/distal-in.

The transducer recordings for each load represented in Appendix I are corrected for the six possible measuring device displacements. The correction values are obtained from force system vs load plots obtained by displacing the attachment receptacle with the facebow in position, and recording the resulting change in the force system components. An example of a typical correction table for a single sample is shown in Table V of Appendix I.

Appendix II contains the corrected force system values. Tables I and II are the rigid attachment data, Tables III and IV display the non-rigid attachment data. Plots of applied load vs the corrected values of moment-1, lateral force, moment-2, and the distal force appeared linear in nature. For both the rigid and non-rigid attachment experiments this relationship was evident. Evaluating this relationship, a linear regression analysis was performed with applied load selected as the independent variable and the individual measurements as the dependent variables. A table displaying the regression coefficients (slopes), intercepts, and correlation coefficients for the facebows tested is found in Appendix III. Graphic representations of the facebow regression lines are illustrated in Appendix III also.

The strength of the correlations indicates very little dispersion of the data about the regression lines. This demonstrates a high degree of accuracy in the measuring capability of the experimental instrumentation. The elevations and slopes of the regression lines plotted indicates Facebow 1 and Facebow 2 are the same. For verification a comparison of the regression lines is made using an analysis of covariance. The F-Test is used in conjunction with the analysis to indicate if the variation between the facebow means is greater than the variation within the facebow means. The table of regression line comparisons is shown at the end of Appendix III.

The analysis demonstrates that in all instances, save for the distal force of the rigid attachment, a statistically significant difference exists between the slopes of the facebows. The elevations of the regression lines are not significantly different. Consideration of the variance about the regression line reveals why the regression lines are statistically different. The correlation coefficients indicate a strong association between applied load and the force or moment measured for both facebows. The regression lines are very close in elevation and slope but the analysis of covariance determines them to be two distinct facebow populations. There is little dispersion about each regression line. Statistically the facebows are different; from a practical standpoint they are not. To illustrate this, refer to the non-rigid attachment plot of applied load vs moment-2. The samples illustrated display the greatest divergence of the data. With a load of 500 gms the predicted value for Facebow 1 is 1,153.0 gm-mm. For Facebow 2 the value is 941.5 gm-mm. The percent variation between the moments is:

$$\frac{1,153.0 - 941.5}{1,153.0} \times 100 = 18.3\%$$

With a variation of only 18.3% at the extreme of the loads tested the facebows, for all practical purposes, can be considered the same.

The statistical difference in slopes may be attributable to error in the calibration constants assigned to the individual transducers. Independent machine calibrations were used for each facebow. Any inaccuracy could produce erroneous force system magnitudes. The effect of an inaccurate calibration constant is compounded by the fact that the correction values for measuring device deflections would also be in error.

There may have been small geometric descrepancies between the facebows. Although the configurations appeared identical the large wire cross-sections and associated high load deflection rates make the force system sensitive to even small configuration differences.

Having concluded the facebows tested in the rigid attachment were the same the data was pooled. Regression coefficients, intercepts, and correlation coefficients indicative of the population were calculated. A similar procedure was performed with the facebows tested in the non-rigid attachment experiments. A table of these values is found in Appendix IV. Plots of the regression lines are also found in Appendix IV. Visual examination of the regression lines reveals similar slopes and elevations of the two groups. The analysis of covariance was again employed to test the hypothesis of equivalence between the rigid and non-rigid attachment data. A table of comparisons is included at the end of Appendix IV.

With the exception of moment-2, the elevations of the regression lines show no significant difference. However the slopes are all statistically different. The arguments of minimal dispersion about the regression lines, calibration constant inaccuracy and configuration descrepancies are again cited as possible sources of divergence.

B. Material Properties and Analysis

Two samples of raw data appear in Figure A and Figure B of Appendix V. The relationship between the load and change in length of .045" and .072" wires are plotted. The modulus of elasticity and yield strengths were calculated using the relations described in Methods and Materials (page 26). Specific calculations pertaining to Figures A and B are included in Appendix V following the plots. The results of the testing appear in Tables I and II of the Appendix.

Comparing the values of the modulus of elasticity and yield strengths between lot #1 and lot #2, no practical difference was evident. The descrepancies in magnitude are slight and most probably reflect discrete differences in alloy composition between lots as well as experimental measurement variation.

The higher modulus of elasticity values observed following the heat treatment are consistent with the reports in the literature. The most dramatic change between the as-received and heat treated values was found in the .045" wire. Comparing the modulus means a

+3.5 x 10^6 psi difference was noted in lot #1 while in lot #2 a change of +2.0 x 10^6 psi difference was recorded. The same heat treatment performed on the .072" wire yielded smaller gains in modulus of elasticity magnitude. A comparison of means reveals a +1.4 x 10^6 psi difference in lot #1 while a negligible change was recorded in lot #2. Alloy response may be related to the change in modulus of elasticity values for the different wire cross sections. The .045" wire is a 304L steel (low carbon steel), whereas the .072" wire is a 455 steel.

All of the wires, with the exception of the .072" in lot #2, experienced a reduction in yield strengths subsequent to the heat treatment. This observed reduction was expected because the heat treatment decreases the dislocation density of the wire.

A dramatic reduction in yield strengths was observed in lot #1 following the recrystallization heat treatment. Generally, the lowest yield strengths are attained following an annealling process such as this. The recrystallization heat treatment was more a test of the instrumentation procedure than providing any information to the theoretical analysis.

The values for the heat treatment wires of lot #2 were adopted for the theoretical analysis because the facebows configured for the experimental comparison were from this lot. The modulus of elasticity for the .072" samples was 27.68 x 10^6 psi and 26.13 x 10^6 psi for the .045" samples. These are 2.88% and 6.68% lower than the commercially accepted values of 28.5 x 10^6 psi for the .072" wire and

28.0 x 10^6 psi for the .045" wire respectively. The yield strength for .072" wire was 238,242 psi and 181,430 psi for the .045" wire. The value for the .045" is slightly lower than the accepted 184-210,000 psi range, while the .072" value is higher than the 195-220,000 psi range generally quoted.

II. THEORETICAL

Table I of Appendix VI is the theoretical values obtained from the finite element analysis which assumes a rigid attachment. The theoretically deformed shape of the facebow loaded bilaterally with 500 gms is illustrated in Figure 12. The forces appearing are those on the wire. The direction of the forces and moments produced are equivalent to those measured experimentally.

To allow comparison between experimental and theoretical models linear regression lines were derived from the theoretical values found in Appendix VI. Plots of the linear regression lines for the theoretical and the experimental results were constructed and are found in Appendix VII. Substantially higher values for the theoretical model than the experimental were observed for any given load, excluding distal force where the theoretical and experimental values were in agreement. The loads to the outer bows in the experimental series were applied within plane, confirmed by the symmetrical distal forces.

The plots reveal the most extreme divergence of the regression lines at the highest applied load, 500 gms. At this loading the theoretical values predicted from the linear regression equations for the lateral force, moment-1 and moment-2 are 37.2%, 45.3% and

37.0% higher respectively than those predicted by the experimental linear regression equations. A number of probable causes can be postulated for this descrepancy. Deflection of the members of the measuring device, specifically the cantilever beams and torsional elements, may not have been entirely accounted for by the correction factors. Although every attempt was made during the correction procedure to load the attachment receptacles at the center of rotation, a slight deviation from the center could introduce error.

Another contributing factor to be considered is the press-fit junction of the .045" wire with the inner arch tube. This complex joint may not have responded analytically as it did experimentally. More information about this interface may be required before accurate deflection is predicted. The mathematical model could possibly be inaccurate and in need of modification, also.

The modulus of elasticity of the .045" inner arch tube could not be tested experimentally and the advertised value was employed. This value may not be representative and could contribute error if the modulus of elasticity was lower than the commercially accepted value.

The effect of allowing the attachment point of both inner arms to rotate through 1 degree dramatically affected the magnitude of the moments and lateral forces produced by the facebow. The values in Table 1 of Appendix VII are the force systems produced as attachment rotation was varied. A plot illustrating the effect is also included in Appendix VII. A rotation of only .6 to .8 degrees effectively negates the moments. The lateral force has also dropped dramatically suggesting that this force is intimately related to the angular deflection of the facebow.

Understanding that small amounts of attachment rotation has profound effects on the moments and lateral forces, one can appreciate how accurate the corrections for the deflections of the measuring device must be. Assuming that incomplete correction to the angular displacement transducer in the experimental device was cause for the descrepancy between the theoretical and experimental force values, the amount of rotation necessary to complete the correction can be calculated from the plot on attachment rotation (Appendix VII). A rotation of .33 degrees for moment-1, .25 degrees for moment-2 and .4 degrees for lateral force could effectively eliminate the difference in force system. Clearly such small displacements make it imperative that a rigid measuring device be employed when testing appliances with high load deflection rates.

The effect of choosing an attachment point 7 mm distal to the original attachment point at the molar offset revealed a reduction in moments and lateral forces of approximately 30%. Table II of Appendix VII shows the force system values.

III. EXPERIMENTAL - THEORETICAL CORRELATIONS

Tables I and II of Appendix VIII are the uncorrected experimental results from the divergent arch in its original orientation and its inversion, respectively. Tables I and II of Appendix IX show the corrected experimental values for the divergent arch. The correction values were obtained from force system vs load plots obtained by displacing the attachment receptacle with the divergent arch in position, and recording the resulting change in force system components.

The force system values recorded in Appendix IX for the divergent arch in its original and inverted orientation appear very close. The values for the moments and lateral force are considerably higher than those obtained with the facebow, this probably owing to the absence of a rigid solder joint in the anterior of the divergent arch. The data collected appeared linear in nature and thus linear regression analysis was applied. A table of regression coefficients, intercepts and correlation coefficients is recorded in Appendix X along with plots of the regression lines. The hypothesis that the force systems of the original orientation was equivalent to the inverted was tested by an analysis of covariance. A table of comparisons is found near the end of Appendix X. All of the regression lines showed no significant difference in elevations. The slopes were all stattistically different with the exception of those in moment-1. The same divergent arch was employed for both orientations in the measuring device so no error can be attributed to configuration other than the possibility of an asymmetry in arch form. An asymmetric arch would be reflected in the moment values; moment-1 would be larger than moment-2 in one orientation and when inverted the opposite would hold true. This is not observed in the results. The same rationale would hold if the loading point were slightly offset from the true geometric center of the arch. The magnitudes of the moments would vary with an orientation change in the measuring device. This is not observed. An examination of the correlation coefficients reveals very little dispersion about the regression lines. Although the analysis of covariance shows a statistical difference in the slopes no practical difference exists, making it reasonable to assume the data is the same. The data was pooled and new regression lines derived. A table of the regression coefficients, intercepts and correlation coefficients for the pooled data are found at the end of Appendix X.

Table I of Appendix XI shows the theoretical values obtained from the finite element analysis. A computer graphics display of the deformed divergent arch is illustrated in Figure 13. A centrally located load of 500 gms is depicted.

The theoretical values were subjected to linear regression analysis. The regression lines and the pooled experimental results were plotted. These plots are found in Appendix XII. Comparing the moments, the experimental values are slightly lower than the theoretical analysis would predict. The plots indicate the most extreme divergence of the moment regression lines occurs at the highest applied load. At 1000 gms load the theoretical values predicted by the equations for moment-1 and moment-2 are 26.3% and 24.0% higher respectively than those predicted by the regression equations for the experimental data. The lower experimental moment values may be related to the calibration constants chosen for the angular transducers. Experimental procedure used to determine correction factors is also suspect. Incomplete correction of the angular transducers could account for the descrepancy between the experimental and analytical data.

The lateral force and distal force values are slightly lower in the experimental series. From a practical standpoint they are equivalent to the analytical values.

DISCUSSION

The rigid and non-rigid attachment investigations indicate a strong association between applied load and the force systems measured, as revealed by the correlation coefficients derived for each regression line. With a high degree of certainty it can be postulated that as the load to the outer bows of a symmetrical facebow is increased, an associated increase in the distal forces will occur. Lateral forces tending to expand the bucco-lingual arch dimensions, as well as moments tending to rotate the teeth mesial-out/distal-in, will increase as a function of load. From a clinical standpoint the difference in force system magnitude between the rigid and non-rigid attachment series is negligible. In general, the predicted values obtained from the regression equations for the non-rigid attachment are lower than the predicted rigid attachment values. This is consistent with the assumption of relaxing the deflection of the wire within the tube of the non-rigid attachment. However, the overall reduction in force system magnitude for a .045" wire in a .051" tube is clinically insignificant. A looser wire-tube interface would probably be required to reduce the force system magnitude appreciably.

The strength of the correlation coefficients observed in the experimental data indicates that the uniplanar displacement transducer is an accurate device for measuring force systems. However, deflection of the attachment receptacles invariably alters the geometry of the appliance tested. The large cross-section of the wires forming the facebow contribute to a high load deflection rate, making the appliance highly sensitive to small geometrical changes. To avoid erroneous force system measurements it is imperative that a rigid measuring device be employed when testing appliances of large cross-section, high modulus materials.

The experimentally determined values for the modulus of elasticity following heat treating the facebow wires are slightly lower than the commercially accepted values. Using these values as input to the theoretical analysis produces force systems lower in magnitude than if higher commercial values were employed. The heat treatment appears to effect a greater change in the modulus of elasticity in the .045" wire than the .072" wire. This may be characteristic of the alloy's structural response to the heat treatment performed. The .045" wire is a 304L steel, while the .072" wire is a 455 steel. Further research in this area of heat treatment of stainless steel wire may yield more definitive reasons for the changes in modulus values.

The theoretical analysis performed on the facebow produces force systems similar in direction to the experimental series, but larger in magnitude. The descrepancy may be attributable to errors associated with correcting for deflections of the experimental measuring device. The junction of the .045" wire with the inner arch tube forms a complex joint that requires additional study to determine the deflection characteristics. Future theoretical analyses of these complex multiple beam structures should consider the interfaces between the members of the appliance. Modification of the mathematical model may be necessary to more closely approximate the experimental results. However, before revising the analytical model it is advisable first to assure the experimental instrumentation is entirely accurate. A measuring device with minimal deflection should be designed for use with high load-deflection appliances. Results from such a device could be compared with those from the theoretical model and appropriate revisions could then be considered if a descrepancy still exists.

The theoretical analysis indicates the moments and lateral forces produced by the facebow are dramatically affected by permitting rotation of the attachments in the direction of the moments. Negation of the moments is accomplished with only .6 to .8 degrees of rotation. Because of the high angular load-deflection rate the facebow resists any large moment applied to the tooth attempting to rotate it. Clinically this has significance because the facebow can create a moment with the distal force acting buccal to the center of resistance of the tooth of attachment. This moment tending to rotate the tooth mesial-out/distal-in will rotate the tooth until the facebow resists the tooth rotating. Lingual appliances such as the palatal arch employed to deliver moments in the occlusal plane may be prevented from rotating a tooth if a facebow is worn simultaneously. The facebow may allow only a small degree of rotation but will inhibit further rotation once the moment from the palatal arch falls below the resisting moment from the facebow. Headgear therapy may have to be interrupted if rotation of attachment teeth are being rotated with other mechanotherapy. The same holds true for changes in the bucco-lingual dimensions. The facebow could effectively prohibit lateral movements produced from other appliances in the arch.

The lateral forces produced are closely related to the angular deflection of the facebow. This force rapidly dimishes with a small amount of bracket rotation. Using a passive palatal arch to prevent the teeth from rotating or changing in bucco-lingual dimension during headgear therapy is unnecessary, at least with this particular face-bow geometry. The force system produced by the facebow, the moments and lateral forces, are of sufficient magnitude to move teeth but the deflection of the facebow will prevent any appreciable tooth movement. The effect of varying the attachment point also affects the force systems. The clinical practice of adding washers to the inner bow terminals, thereby extending the facebow anteriorly, creates a new attachment position and consequently a new force system. The moments and lateral forces will tend to be lower in magnitude.

With the simple divergent arch, the force system values of the experimental approach those of the analytical investigations. From a practical standpoint, only the moments predicted by the theoretical analysis are higher than the experimental values obtained. The descrepancy in moment values is considerably less than a similar comparison of facebow data. The absence of any structural joints, and the maintenance of a constant cross-section of wire appears to have contributed to a closer approximation of force system magnitudes between the experimental and mathematical models.

Of interest is the larger lateral force and moment magnitudes produced with the divergent arch compared to those values from the facebow. Part of this descrepancy can be attributed to the absence of a large solder joint at the anterior of the divergent arch. It can be stated that as the inner bow of a facebow is made more rigid there will be an associated reduction in the lateral and angular deflection of the appliance. From a clinical standpoint, the facebow cannot be made so rigid as to prohibit the manipulation of it. Therefore, it is suggested that the anterior segment of the facebow be made as rigid as possible since this is the area receiving the least amount of adjustment clinically. A broad solder joint appears to be the most expedient means of making the anterior region more rigid without compromising the configuration of the remainder of the facebow.

SUMMARY

The objective of this study was to determine the force systems produced at the inner bow terminals of a facebow during symmetrical loading of the outer bows. Experimental and theoretical analyses were employed to determine the force systems. The experimental investigation utilized a uniplanar displacement transducer system for measurement. Tests included varying the load and comparing the force systems of rigid and non-rigid attachments. The non-rigid attachment series modelled the clinical setting where a wire-tube interface exists. Results of the rigid attachment series were compared to theoretical values derived from a finite element analysis of the same facebow geometry. The effect of bracket rotation and varying the position of bracket attachment were evaluated theoretically also. The material properties of the facebow, specifically the modulus of elasticity and yield strength, were measured experimentally and the values input to the finite element analysis. The divergent arch, an elementary model employing a constant cross-section, was chosen to compare the experimental and theoretical analyses. The following are the significant findings.

1. The facebow, properly loaded, is an appliance capable of delivering bilateral distal forces.
2. In addition to distal forces, lateral forces tending to move teeth buccally are produced as well as bilateral moments rotating teeth mesial-out/distal-in.
3. The force systems produced by the facebow vary linearly as a function of applied loads.

4. Higher values for lateral forces and bilateral moments were observed with a rigid attachment. The lower force system values obtained with a non-rigid attachment relate to the loose wire-tube interface. Looseness permits a small amount of lateral and angular deflection within the tube. The reduction in force system is of little practical/clinical significance.
5. The theoretical analysis demonstrates that the amount of bracket rotation required to eliminate the moments produced by the facebow is minimal. With a load of 500 gms applied to this particular facebow geometry the bilateral moments (1800 - 2200 gm-mm) were removed with .6 to .8 degrees rotation. There appears to be no clinical need to accomodate the moments produced by the facebow.
6. The lateral force tending to move the teeth buccally diminishes quickly as the bracket rotates. The lateral forces, created by angular deflection of the facebow, are insignificant and do not require control clinically.
7. The facebow can effectively prevent rotation or lateral movements of teeth from moments or forces applied by other mechanotherapy.
8. The theoretical analysis demonstrates that positioning the attachment point distal to the molar stop reduces the lateral forces and moments. The clinical practice of adding washers to the ends of the inner bow wires to advance the facebow anteriorly will alter the force system delivered.
9. The force systems produced by the facebow are highly sensitive to the configured geometry. The high load-deflection characteristics relating to the large cross-sections of high modulus steel

wire are responsible for the sensitivity.

10. Statistical evaluation reveals the uniplanar displacement transducer system to be a very precise device for measuring force systems produced by orthodontic appliances. To eliminate the procedure of correcting for deflection of the cantilever beams and torsional elements, a more rigid apparatus should be designed for testing high load-deflection rate appliances.
11. The values determined experimentally for the simple divergent arch approach those obtained by the finite element analysis. The small descrepancies in force/moment magnitudes are probably related to incomplete correction of the measuring device for deflection, specifically angular deflection.
12. The finite element analysis can be used to test force systems produced by various appliance geometries if the physical properties of the wires are known. Modification of the mathematical model would be appropriate if investigation with a rigid experimental measuring device maintained the analytical-experimental descrepancy.
13. Attention to the deformation characteristics at the junction of complex beam structures may assist in the prediction of accurate force systems that correlate with the experimental investigations.
14. If minimal lateral and angular deflection is desired in a facebow, a commercial unit with a broad anterior reinforcing solder joint is recommended.

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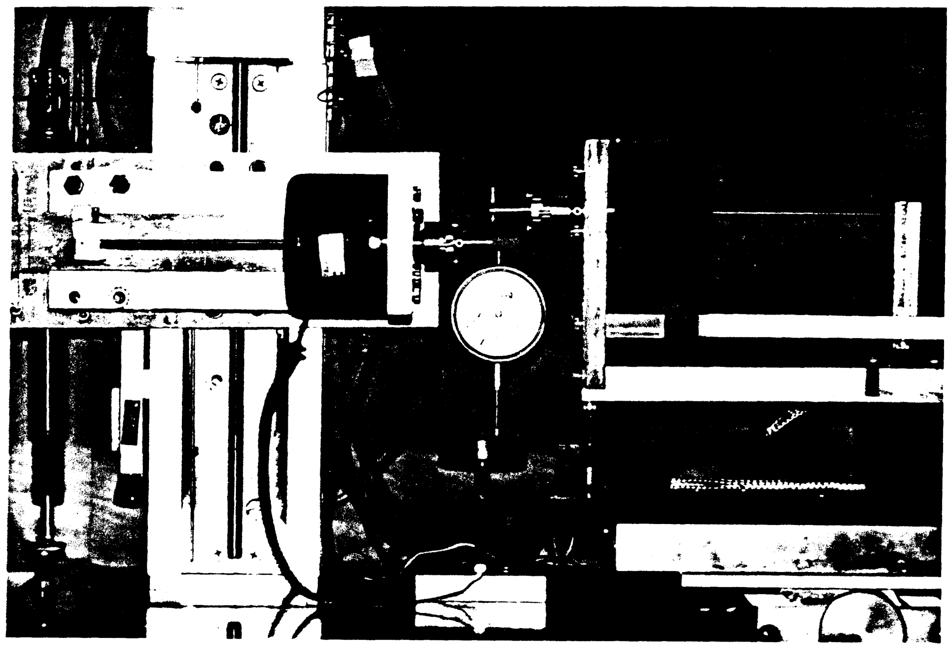


FIGURE 1. The uniplanar displacement transducer system employed in the experimental investigations.

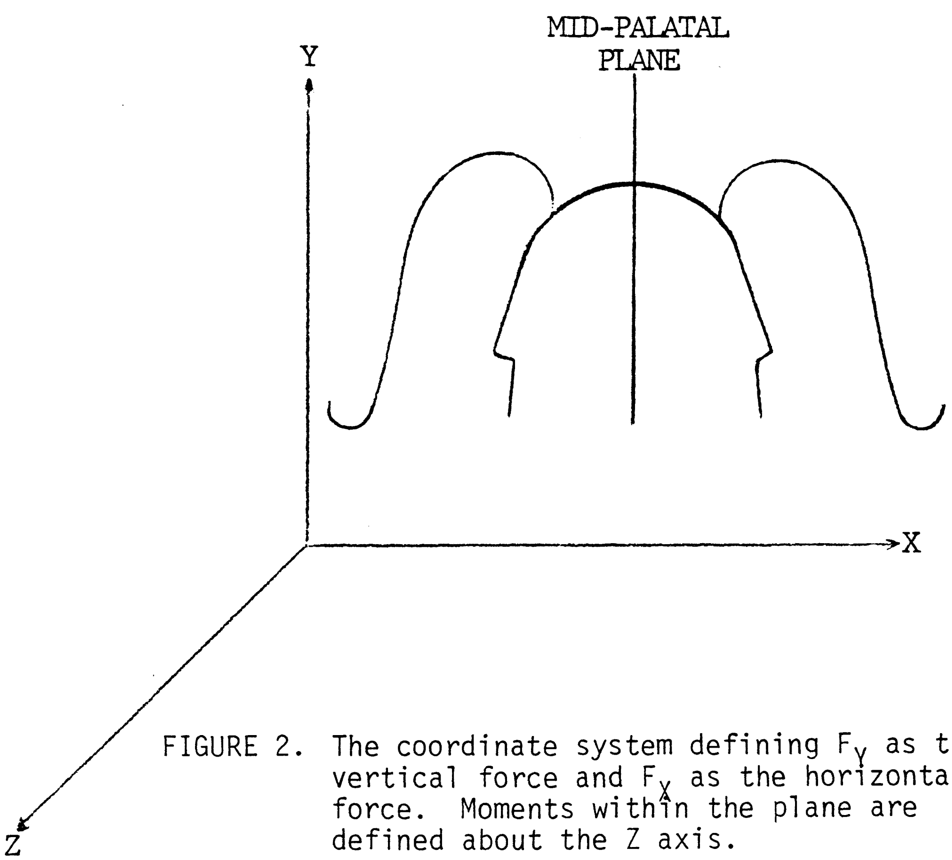


FIGURE 2. The coordinate system defining F_Y as the vertical force and F_X as the horizontal force. Moments within the plane are defined about the Z axis.

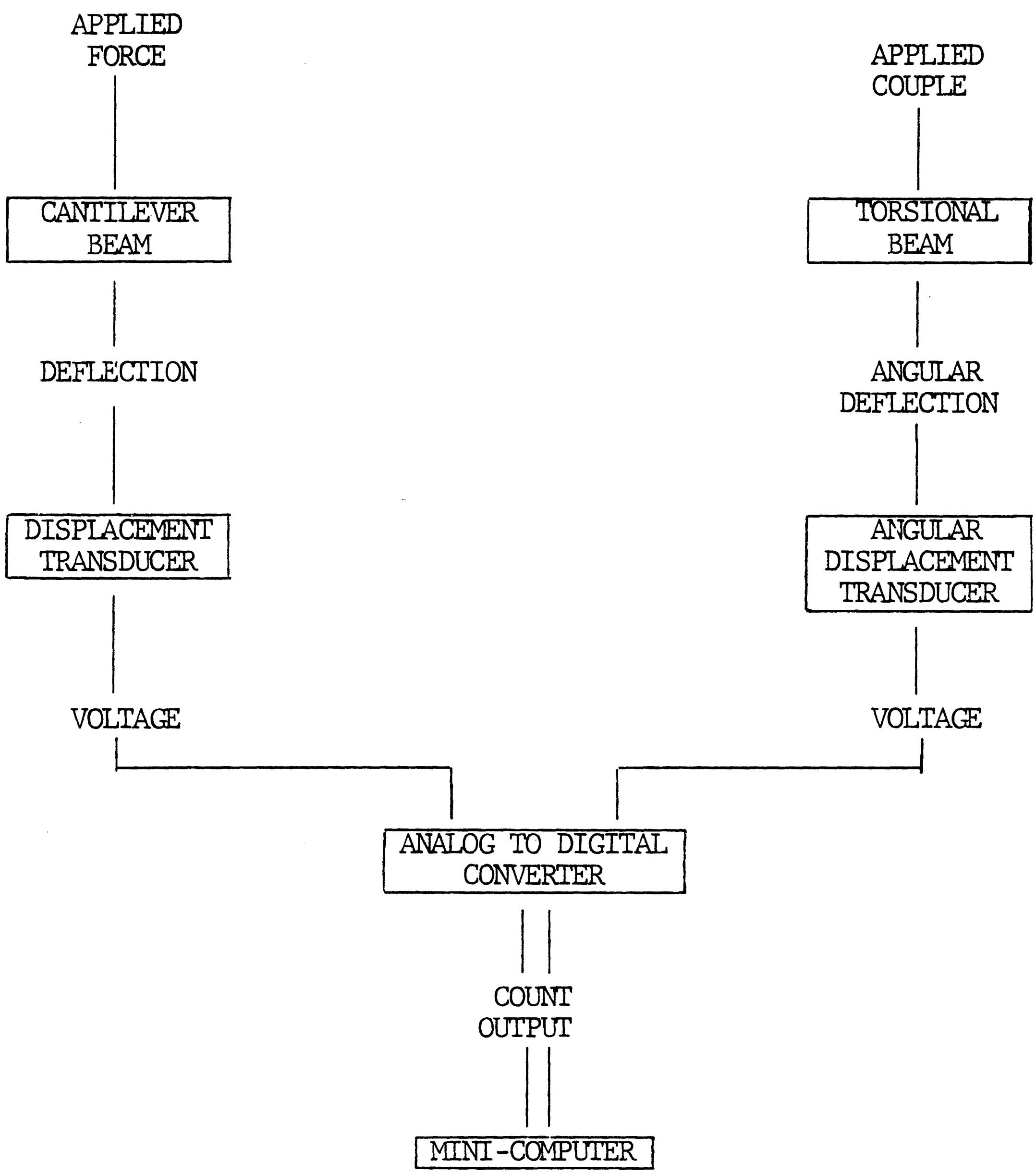
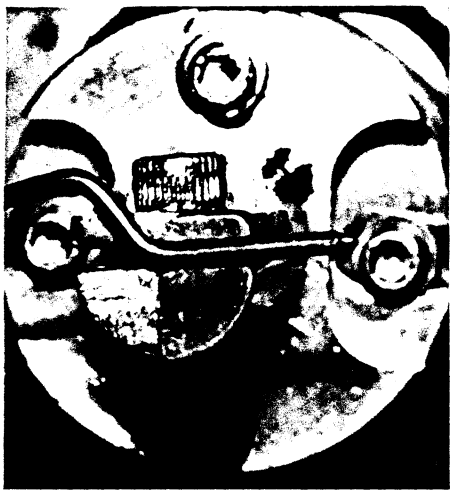


FIGURE 3. Flowchart depicting transduction of mechanical energy to count outputs.



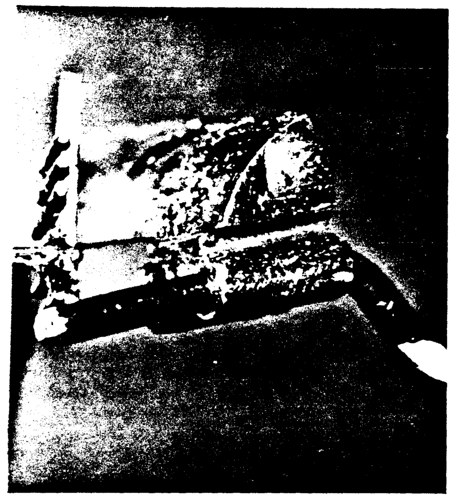


FIGURE 4.

A
Rigid Attachment

B
Non-rigid Attachment

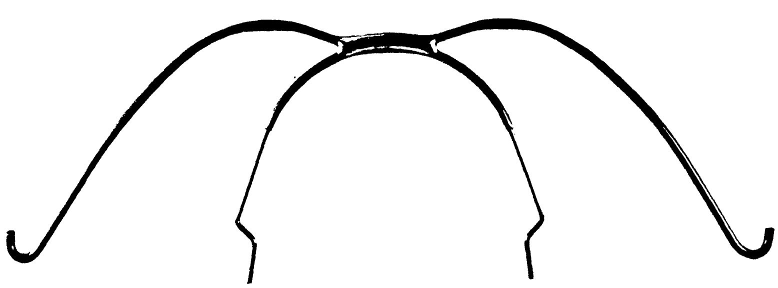


FIGURE 5. The geometry of the configured facebow. This geometry was maintained for the experimental and theoretical investigations.

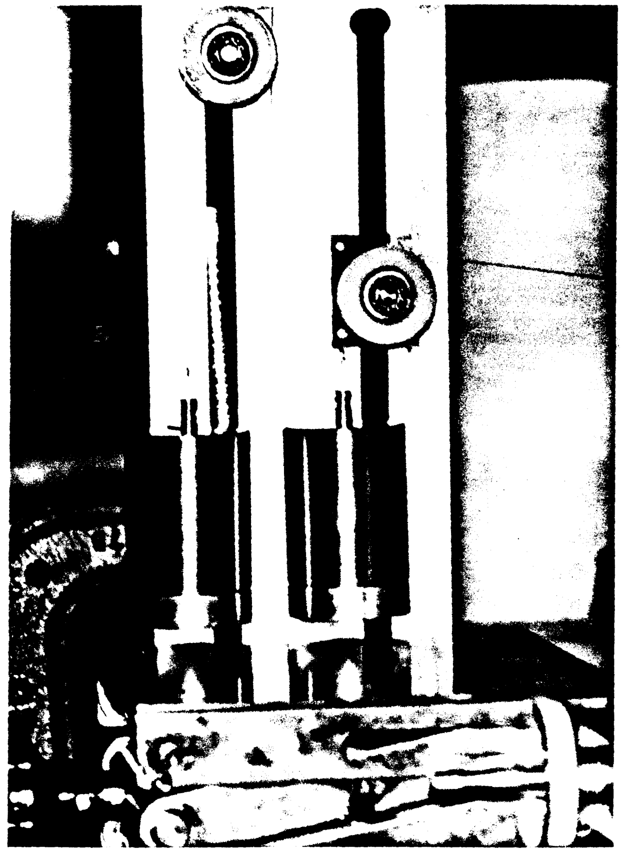


FIGURE 6. Loading device and elevating platform used for synchronous loading of facebow in the non-rigid attachment experiments.

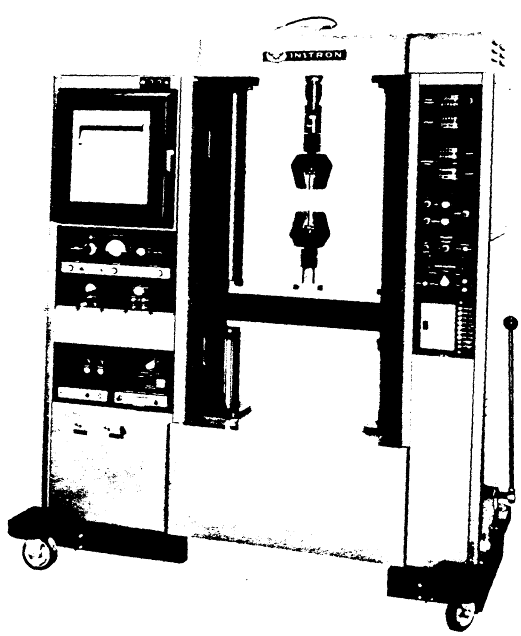


FIGURE 7. The Instron Universal Testing Machine

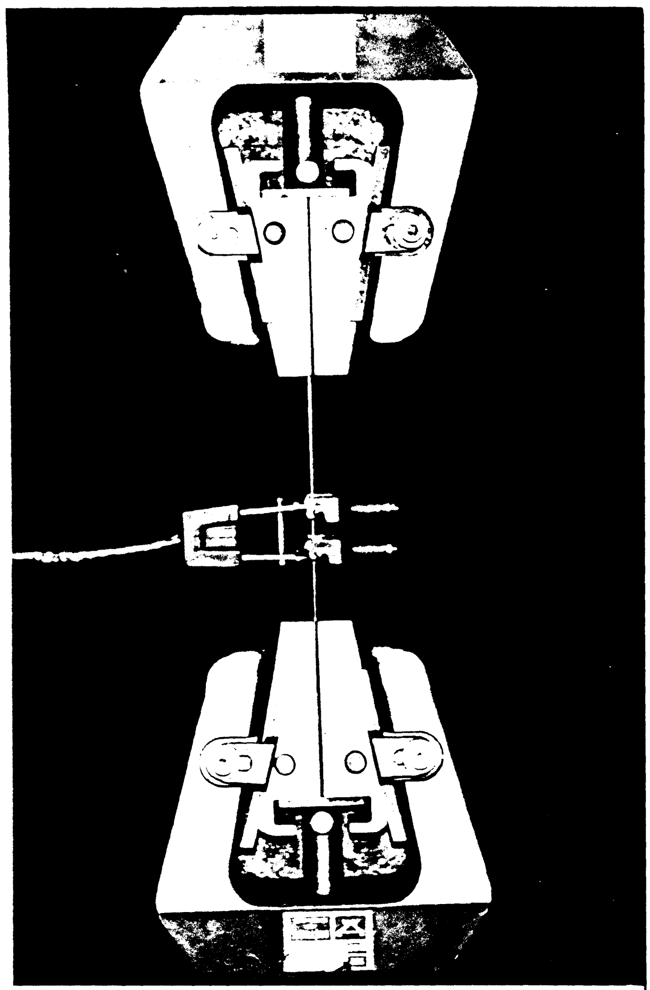


FIGURE 8. A wire sample secured in the grips of the Instron device. The extensometer attached to the wire sample measures the change in length.

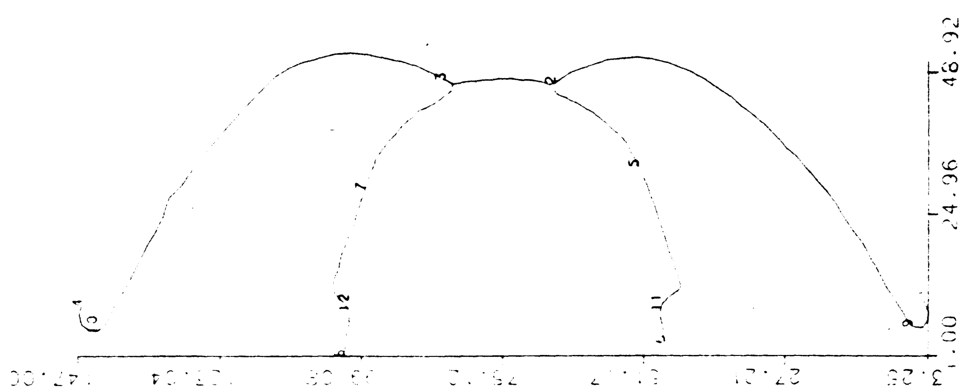


FIGURE 9. The geometry of the facebow as reproduced by the computer in the finite element analysis

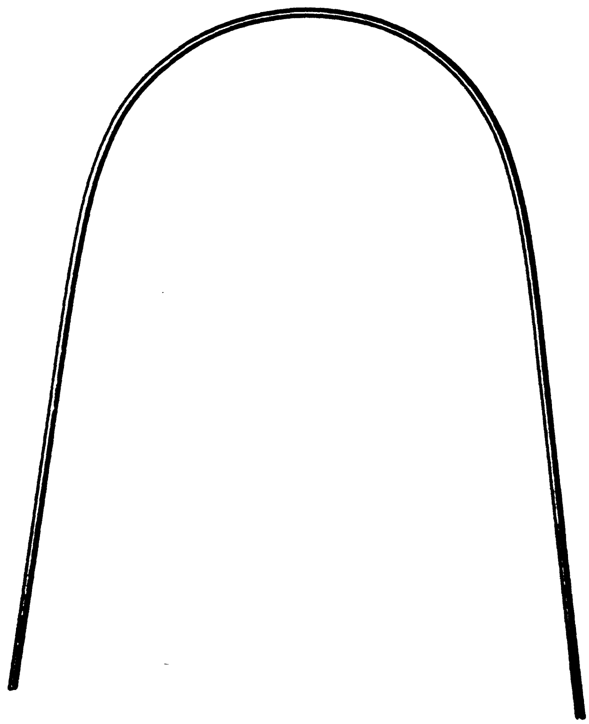


FIGURE 10-A. Geometry of the divergent arch

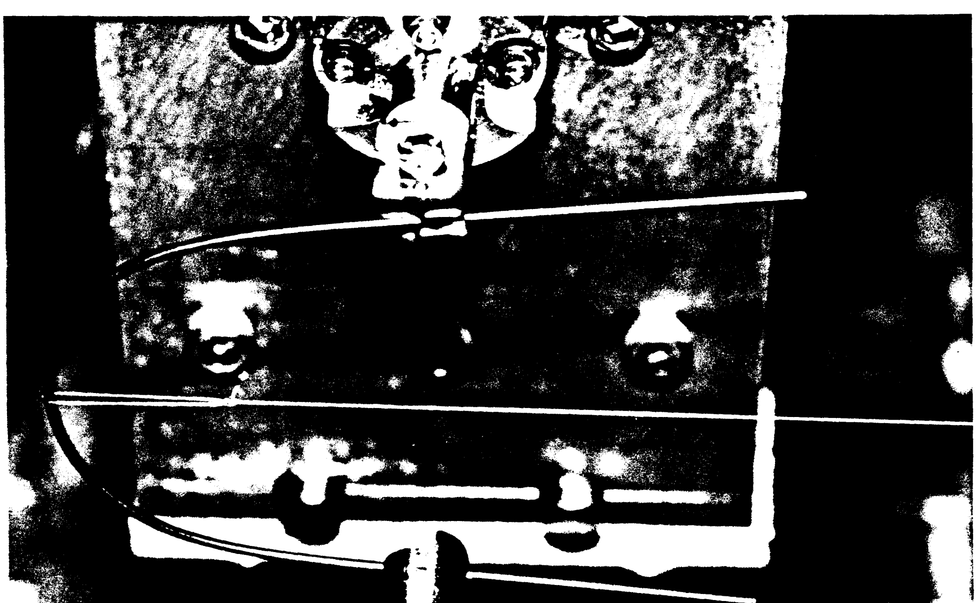


FIGURE 10-B. Divergent arch rigidly fixed.

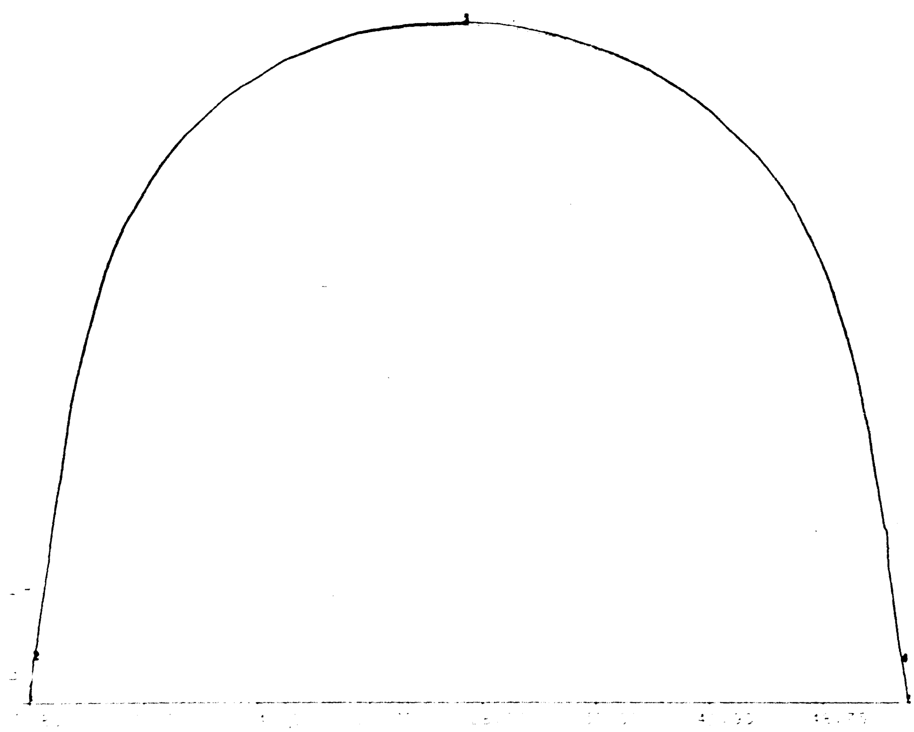


FIGURE 11. Geometry of the divergent arch as reproduced by the computer for the finite element analysis.

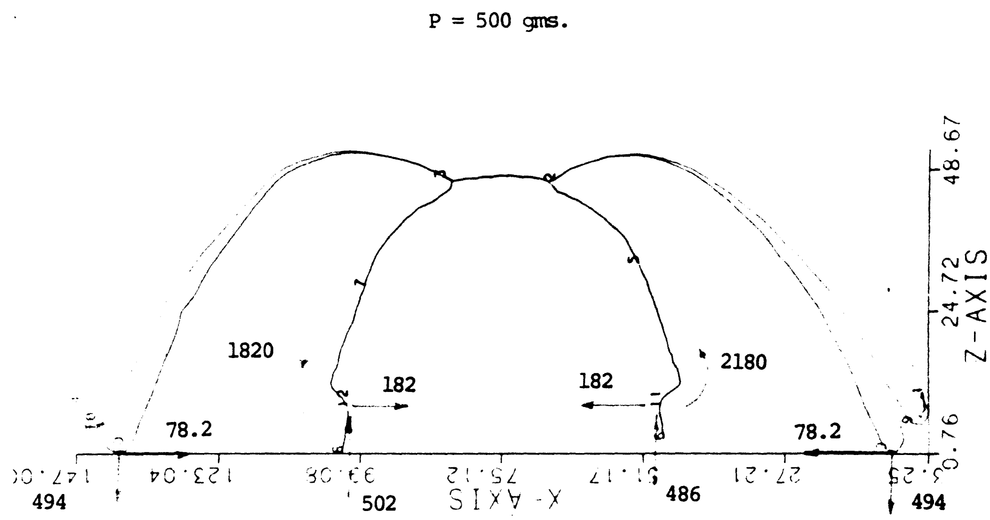


FIGURE 12. The theoretically determined deformed shape of the facebow from a bilateral load of 500 grams.

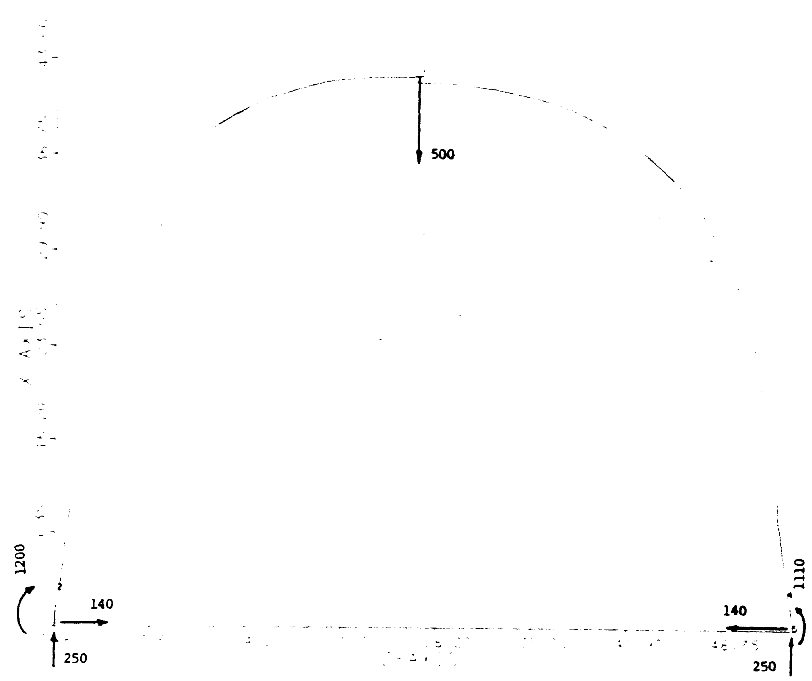


FIGURE 13. The theoretically determined deformed shape of the divergent arch from a single point loading of 500 grams.

APPENDIX I

TABLE I

FACEBOW I - RIGID ATTACHMENT

5 SAMPLES

UNCORRECTED FORCE SYSTEM VALUES

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
100	-243.84	19.63	170.07	-93.74
200	-443.64	35.64	307.83	-189.77
300	-631.32	51.21	406.10	-289.09
400	-821.76	64.69	525.02	-379.70
500	-1094.88	83.16	631.97	-491.54
100	-238.08	20.64	181.29	-92.70
200	-432.48	35.53	309.29	-189.11
300	-617.88	49.44	406.66	-286.40
400	-840.12	64.14	508.09	-385.77
500	-1065.72	80.88	624.71	-485.39
100	-243.84	18.83	180.48	-93.90
200	-423.84	33.32	304.30	-190.11
300	-629.40	49.66	406.60	-284.39
400	-820.80	64.32	522.97	-374.17
500	-1105.08	81.60	628.00	-489.62
100	-240.36	19.78	161.82	-94.30
200	-429.12	35.53	304.54	-188.58
300	-620.40	48.53	397.17	-288.75
400	-843.84	64.76	518.75	-387.48
500	-1076.28	81.49	633.52	-483.27
100	-232.68	19.74	175.96	-92.21
200	-435.80	36.00	304.17	-190.74
300	-639.72	49.95	398.68	-298.50
400	-831.00	65.19	516.77	-382.91
500	-1098.84	82.72	634.20	-491.02

TABLE II

FACEBOW II - RIGID ATTACHMENT

6 SAMPLES

UNCORRECTED FORCE SYSTEM VALUES

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
100	-202.20	17.78	155.31	-96.56
200	-377.04	29.48	276.83	-194.87
300	-551.76	42.41	374.17	-299.83
400	-735.96	57.26	470.95	-395.94
500	-923.40	70.99	585.40	-494.36
100	-207.96	19.49	164.11	-96.59
200	-379.80	30.28	284.83	-193.50
300	-529.56	42.77	378.45	-291.18
400	-722.40	54.69	476.97	-395.80
500	-928.80	70.99	586.77	-489.70
100	-205.56	17.86	167.59	-94.22
200	-382.44	30.50	287.56	-191.79
300	-533.88	40.57	379.25	-290.96
400	-725.64	54.94	489.92	-388.23
500	-932.40	69.68	595.26	-490.09
100	-204.48	17.46	171.80	-96.08
200	-374.04	30.50	288.49	-193.51
300	-538.08	41.87	384.28	-289.79
400	-715.80	53.57	489.99	-388.03
500	-937.08	69.90	600.10	-489.98
100	-210.00	17.02	165.23	-94.53
200	-378.24	29.34	284.70	-192.87
300	-534.36	41.76	385.45	-290.40
400	-723.24	54.73	477.96	-389.66
500	-929.88	70.84	603.26	-485.70
100	-206.76	17.31	161.63	-94.58
200	-372.84	31.73	290.84	-192.74
300	-528.24	42.38	391.59	-288.16
400	-723.00	54.29	472.94	-385.53
500	-945.12	68.31	592.04	-490.04

TABLE III

FACEBOW I - NON-RIGID ATTACHMENT

10 SAMPLES

UNCORRECTED FORCE SYSTEM VALUES

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
100	-166.44	14.63	155.73	-99.27
	-139.40	13.99	145.26	-99.47
	-169.72	11.82	150.04	-100.47
	-163.03	13.61	148.65	-100.32
	-138.59	12.43	145.38	-98.04
	-149.20	11.82	144.84	-99.90
	-138.48	11.25	142.24	-101.02
	-105.73	13.31	134.61	-98.08
	-154.62	13.21	145.26	-100.96
	-138.24	12.77	144.23	-99.81
200	-351.78	27.19	441.47	-197.84
	-306.24	24.29	319.56	-195.94
	-346.82	24.69	333.11	-194.76
	-361.00	25.40	321.19	-194.41
	-357.66	26.95	302.68	-199.95
	-357.43	23.95	303.68	-199.20
	-367.92	24.86	296.69	-198.55
	-322.84	20.74	251.68	-197.77
	-335.64	20.84	247.02	-202.34
	-347.63	23.41	260.21	-198.21
300	-621.58	39.05	570.70	-303.48
	-620.54	39.96	562.35	-301.23
	-610.40	41.99	557.14	-300.73
	-622.16	40.23	563.01	-300.09
	-606.36	42.57	544.56	-300.51
	-576.15	40.57	533.07	-295.96
	-605.21	40.84	550.67	-295.49
	-562.89	39.76	500.70	-300.23
	-575.58	39.36	526.23	-297.16
	-582.84	39.59	520.29	-300.12

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
400	-828.66	58.31	721.22	-397.90
	-833.04	56.59	702.41	-399.38
	-830.74	55.84	707.73	-395.89
	-854.14	57.73	732.29	-394.60
	-829.58	55.84	680.93	-394.66
	-775.16	55.13	556.42	-395.96
	-820.24	53.54	622.18	-396.16
	-791.19	51.04	578.26	-392.40
	-772.51	51.28	598.95	-391.61
	-765.02	52.02	619.22	-392.00
500	-1034.13	69.90	786.56	-498.15
	-1011.18	67.90	738.10	-491.59
	-980.17	63.54	623.75	-489.34
	-960.33	60.44	590.00	-488.82
	-991.70	64.15	619.28	-483.98
	-1087.74	67.06	653.22	-489.10
	-1031.13	65.50	726.18	-489.28
	-1027.67	60.47	641.24	-486.09
	-1001.73	62.90	586.97	-488.34
	-997.34	61.48	553.33	-486.09

TABLE IV

FACEBOW II - NON-RIGID ATTACHMENT
10 SAMPLES
UNCORRECTED FORCE SYSTEM VALUES

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
100	-148.04	14.05	143.69	-98.46
	-148.51	15.24	157.12	-98.86
	-144.82	14.63	146.05	-99.67
	-147.58	13.58	137.52	-97.97
	-149.31	12.40	138.85	-98.51
	-149.43	13.04	145.32	-99.41
	-149.66	13.75	132.92	-98.37
	-149.20	13.45	147.01	-99.57
	-149.66	12.63	120.33	-99.83
	-148.97	14.66	115.31	-99.08
200	-294.13	24.90	314.84	-191.25
	-295.63	25.03	281.45	-192.92
	-297.82	22.60	269.95	-192.93
	-294.71	24.05	290.22	-194.50
	-294.36	23.99	247.93	-192.55
	-287.44	23.68	264.20	-192.73
	-288.71	23.04	247.02	-192.34
	-281.91	21.32	234.38	-192.10
	-296.78	23.28	229.78	-191.17
	-277.87	23.24	332.63	-194.41
300	-483.68	38.92	524.11	-291.96
	-517.24	36.96	487.99	-290.41
	-515.39	36.15	461.31	-289.55
	-506.51	39.90	580.44	-291.29
	-506.51	36.79	423.20	-289.40
	-496.02	40.67	580.01	-291.24
	-508.93	39.42	597.80	-290.77
	-450.48	38.92	544.32	-291.09
	-484.84	38.71	411.46	-287.54
	-486.57	35.17	372.01	-288.04

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
400	-712.90	49.05	513.16	-381.17
	-708.75	47.97	508.08	-378.67
	-699.99	47.16	490.05	-377.09
	-700.68	49.02	479.46	-377.20
	-701.37	46.82	425.56	-377.48
	-703.10	46.75	435.42	-378.01
	-693.53	46.69	458.53	-375.58
	-692.84	43.65	387.68	-377.37
	-693.64	48.55	398.39	-377.09
	-699.99	44.52	401.96	-378.48
500	-941.54	62.67	538.27	-476.61
	-951.80	60.34	526.17	-476.85
	-945.34	59.29	507.23	-478.12
	-938.66	62.40	502.39	-474.22
	-941.89	62.70	508.02	-474.89
	-941.89	62.56	535.67	-471.93
	-956.30	59.69	502.09	-479.48
	-942.00	60.77	496.04	-471.96
	-942.35	59.73	512.50	-469.84
	-933.70	61.52	514.31	-466.41

TABLE V

CORRECTION TABLE FOR A SINGLE SAMPLE

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
100	-243.8	19.6	170.0	-93.7
	78.0	0.0	66.0	X (LEFT)
	-58.0	0.5	-44.0	X (RIGHT)
	-40.0	4.2	X (RIGHT)	0.0
	X (LEFT)	4.3	36.5	0.0
	-3.5	X (LEFT)	10.0	0.0
	-21.0	X (RIGHT)	14.5	0.0
CORRECT VALUES	-288.3	28.6	253.0	-93.7

APPENDIX II

TABLE I

FACEBOW I - RIGID ATTACHMENT

5 SAMPLES

CORRECTED FORCE SYSTEM VALUES

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
100	-279.5	28.6	239.4	-93.7
	-276.9	29.7	251.0	-92.7
	-282.0	27.9	249.5	-93.9
	-275.5	28.9	231.1	-94.3
	-269.3	28.9	243.1	-92.2
200	-539.7	53.4	502.2	-189.8
	-528.0	53.3	501.6	-189.1
	-519.2	51.1	498.6	-190.1
	-523.2	53.3	494.9	-188.6
	-533.1	53.8	498.1	-190.7
300	-781.0	79.3	725.8	-289.1
	-764.7	77.6	725.4	-286.4
	-773.8	77.8	722.8	-284.4
	-768.4	76.6	715.3	-288.7
	-796.9	78.1	724.3	-298.5
400	-1015.6	102.6	964.7	-379.7
	-1037.3	102.1	951.8	-385.8
	-1007.3	102.3	955.9	-374.2
	-1044.5	102.7	963.3	-387.5
	-1026.3	103.1	957.1	-382.9
500	-1324.4	131.2	1211.5	-491.5
	-1288.4	128.9	1196.7	-485.4
	-1331.7	129.6	1207.5	-489.6
	-1297.5	129.5	1205.0	-483.3
	-1327.3	130.7	1214.1	-491.0

TABLE II

FACEBOW II - RIGID ATTACHMENT

6 SAMPLES

CORRECTED FORCE SYSTEM VALUES

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
100	-233.5	25.9	232.6	-96.6
	-242.6	27.9	245.1	-96.6
	-236.5	26.3	244.4	-94.2
	-238.3	26.3	249.4	-96.1
	-242.8	25.9	242.3	-94.5
	-237.1	25.7	238.6	-94.6
200	-452.3	44.2	449.3	-194.9
	-456.1	45.2	457.5	-193.5
	-455.5	45.0	459.3	-191.8
	-450.4	45.0	460.7	-193.5
	-453.9	44.0	456.9	-192.9
	-451.5	46.4	463.6	-192.7
300	-686.4	66.4	663.2	-299.8
	-654.1	66.8	658.4	-291.2
	-656.4	64.6	657.1	-291.0
	-662.9	65.9	664.8	-289.8
	-658.6	65.8	664.7	-290.4
	-651.7	66.4	664.7	-288.2
400	-899.5	88.1	875.2	-395.9
	-884.0	85.5	875.6	-395.8
	-881.2	85.8	884.4	-388.2
	-869.6	84.4	882.4	-388.0
	-877.0	85.6	872.5	-389.7
	-880.6	85.6	872.5	-385.5
500	-1108.1	111.1	1103.5	-494.4
	-1107.2	111.1	1104.1	-489.7
	-1110.9	110.0	1110.6	-490.1
	-1116.9	110.9	1116.7	-490.0
	-1106.2	110.9	1117.4	-485.7
	-1121.4	108.4	1107.7	-490.0

TABLE III

FACEBOW I - NON-RIGID ATTACHMENT

10 SAMPLES

CORRECTED FORCE SYSTEM VALUES

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
100	-258.4	22.1	251.7	-99.1
	-231.4	21.5	240.3	-99.3
	-261.7	19.3	246.0	-100.3
	-255.0	21.1	244.6	-100.1
	-230.6	19.9	240.4	-97.8
	-241.2	19.3	239.8	-99.7
	-230.5	18.2	237.2	-100.8
	-196.7	20.4	228.6	-97.9
	-246.6	20.7	241.3	-100.8
	-230.2	20.3	239.2	-99.6
200	-498.8	44.1	632.5	-196.8
	-441.2	39.2	505.6	-195.3
	-481.8	39.9	521.1	-194.4
	-496.0	40.6	509.2	-194.5
	-492.7	42.5	491.7	-199.5
	-492.4	39.5	495.0	-198.8
	-502.9	40.4	487.7	-198.1
	-450.8	36.1	437.0	-202.2
	-463.6	38.1	451.2	-198.1
	-450.8	35.2	441.7	-197.6
300	-678.6	63.4	753.7	-302.7
	-677.5	64.4	745.3	-300.4
	-667.4	66.4	740.1	-299.9
	-679.2	64.6	746.0	-299.3
	-663.4	67.0	727.6	-299.7
	-630.1	64.4	713.1	-295.2
	-662.2	65.0	730.7	-294.7
	-613.9	63.5	682.7	-299.5
	-629.6	63.2	706.2	-296.4
	-639.8	63.6	702.3	-299.3

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
400	-905.7	89.2	1006.2	-397.0
	-910.0	87.5	987.4	-398.5
	-907.7	86.7	992.7	-395.0
	-987.1	89.0	1017.3	-393.6
	-906.6	86.7	965.9	-393.8
	-838.1	84.1	839.4	-395.7
	-887.2	83.2	907.2	-395.8
	-851.2	80.0	865.3	-392.1
	-836.5	80.6	885.9	-391.1
	-829.0	81.3	906.2	-391.5
500	-1211.0	104.2	1282.6	-497.6
	-1189.2	102.1	1227.1	-491.1
	-1134.2	95.7	1100.7	-489.3
	-1116.3	92.7	1065.0	-488.8
	-1149.7	96.3	1106.3	-484.0
	-1251.7	100.2	1145.2	-489.3
	-1203.1	98.9	1216.2	-489.0
	-1191.7	93.4	1117.2	-486.2
	-1157.5	95.1	1064.0	-488.4
	-1151.3	93.2	1030.3	-486.4

TABLE IV

FACEBOW II - NON-RIGID ATTACHMENT

10 SAMPLES

CORRECTED FORCE SYSTEM VALUES

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
100	-358.6	18.2	354.8	-97.9
	-359.1	19.4	368.2	-98.3
	-355.4	18.8	357.1	-99.0
	-358.2	17.8	348.6	-97.4
	-359.9	16.6	349.9	-97.9
	-360.0	17.2	356.4	-98.8
	-360.3	17.9	344.0	-97.8
	-359.8	17.6	358.1	-99.0
	-360.3	16.8	331.4	-99.2
	-359.6	18.9	326.4	-98.4
200	-426.8	34.6	500.6	-190.2
	-424.3	34.4	467.2	-191.7
	-426.5	32.0	455.8	-191.7
	-423.4	33.4	476.0	-193.3
	-419.1	33.2	433.7	-191.2
	-416.1	33.4	450.0	-191.5
	-413.4	32.2	432.8	-190.9
	-406.6	30.5	420.2	-190.7
	-421.5	32.5	415.6	-189.8
	-410.6	32.9	518.4	-193.4
300	-638.8	56.2	764.5	-290.6
	-676.3	55.0	730.4	-288.3
	-674.4	54.1	703.7	-287.4
	-675.6	58.9	822.8	-289.9
	-675.6	54.3	665.6	-287.0
	-665.1	59.7	822.4	-289.8
	-677.9	58.4	840.2	-289.4
	-619.5	57.1	783.7	-290.3
	-639.8	56.0	651.9	-285.4
	-641.7	52.5	612.4	-285.9

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
400	-886.9	72.4	814.9	-377.6
	-882.8	71.4	809.8	-369.3
	-874.1	70.6	791.7	-367.7
	-874.8	72.4	781.2	-367.6
	-866.5	69.7	727.3	-373.6
	-871.2	69.6	737.1	-368.9
	-861.5	69.6	760.2	-371.7
	-857.9	65.2	689.4	-368.5
	-858.7	71.2	700.1	-368.2
	-865.1	67.2	703.7	-369.9
500	-1115.5	91.5	963.3	-467.7
	-1123.8	88.9	951.2	-474.2
	-1119.7	87.8	932.2	-475.4
	-1109.7	90.9	927.4	-471.5
	-1112.9	91.2	933.0	-472.2
	-1119.3	91.4	957.7	-469.4
	-1127.3	88.2	927.1	-476.8
	-1116.4	89.3	918.0	-469.3
	-1116.7	88.3	934.5	-467.2
	-1109.1	90.1	936.3	-463.8

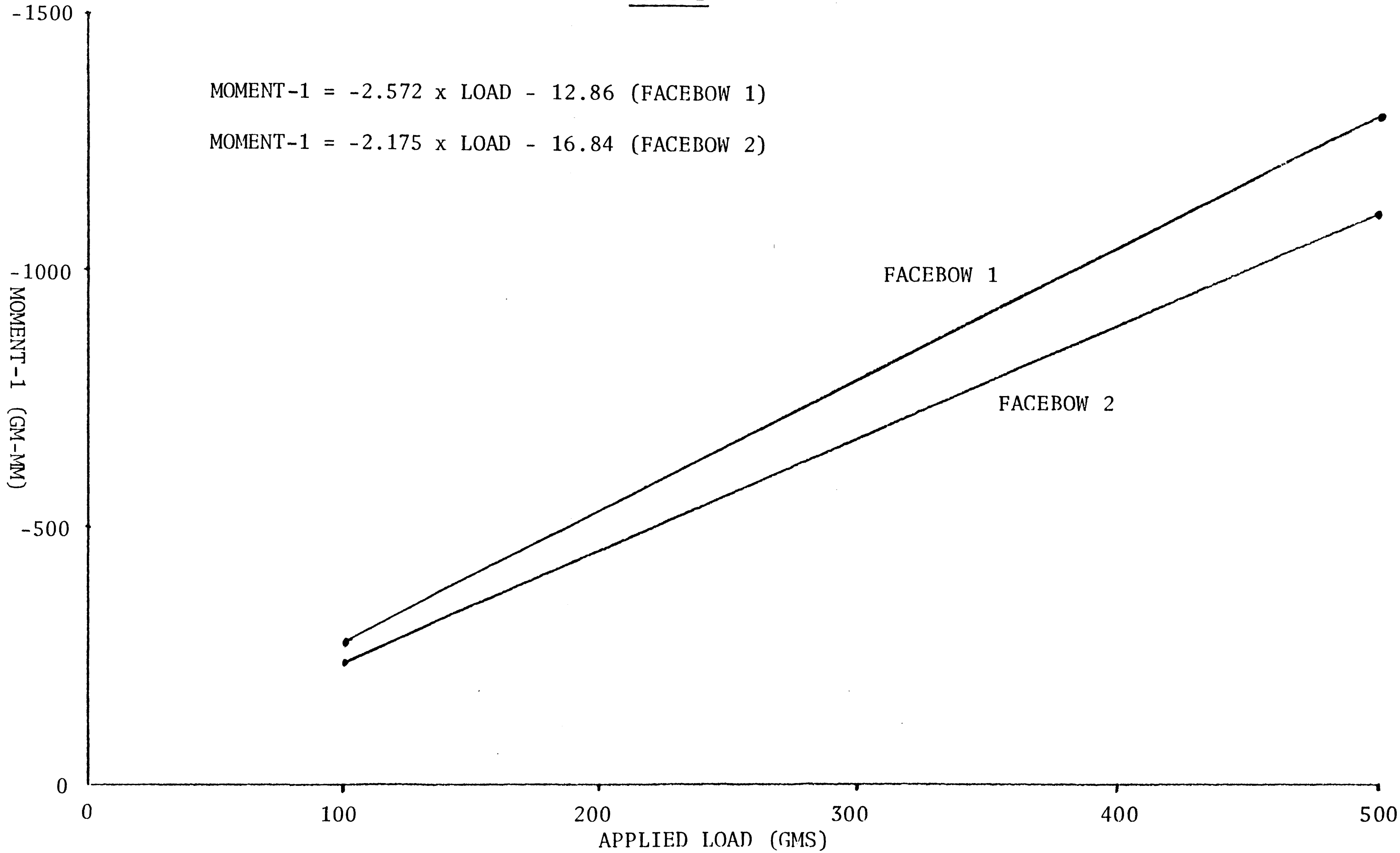
APPENDIX III

TABLE I
REGRESSION COEFFICIENTS (SLOPES),
INTERCEPTS, & CORRELATION COEFFICIENTS
FOR FACEBOWS TESTED IN RIGID AND NON-RIGID ATTACHMENTS

ATTACHMENT	FACEBOW	STATISTIC	MOMENT-1	LATERAL FORCE	MOMENT-2	DISTAL FORCE
RIGID	1	INTERCEPT:	-12.86	2.86	9.80	6.06
		REGRESSION COEFFICIENT:	-2.572	.2519	2.3875	-.9819
		CORRELATION COEFFICIENT:	-.99905	.99951	.99986	-.99952
RIGID	2	INTERCEPT:	-16.84	4.00	23.32	3.75
		REGRESSION COEFFICIENT:	-2.175	.2089	2.155	-.9864
		CORRELATION COEFFICIENT:	-.99956	.99842	.99963	-.9980
NON-RIGID	1	INTERCEPT:	-1.14	1.56	38.40	-3.07
		REGRESSION COEFFICIENT:	-2.28353	.19907	2.22918	-.97587
		CORRELATION COEFFICIENT:	-.99212	.99028	.98594	-.99977
NON-RIGID	2	INTERCEPT	-94.59	-.86	205.67	-6.70
		REGRESSION COEFFICIENT:	-1.96696	.1807	1.47167	-.92362
		CORRELATION COEFFICIENT:	-.98448	.99543	.95413	-.99948

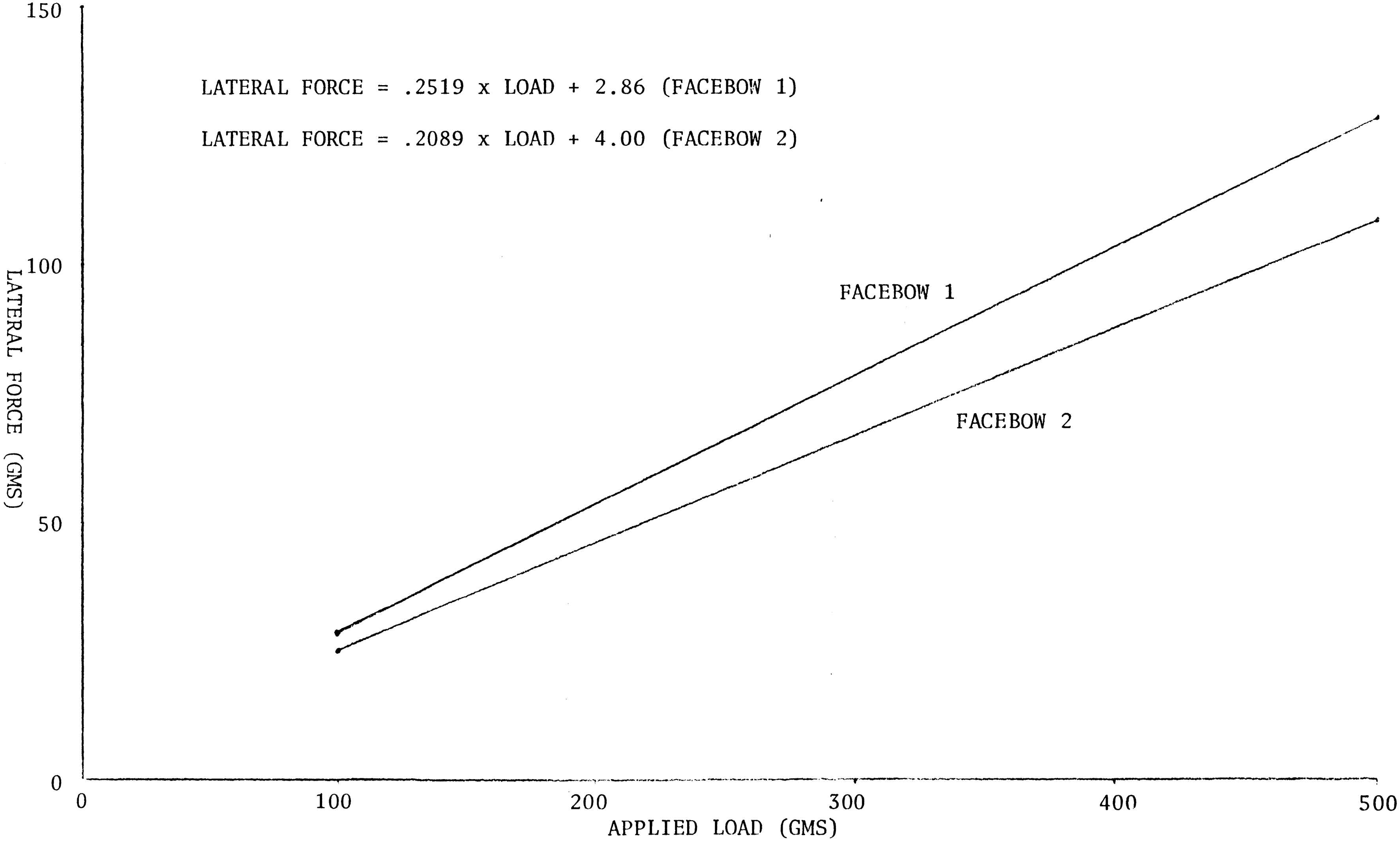
FACEBOW - EXPERIMENTAL - RIGID ATTACHMENT

PLOT I



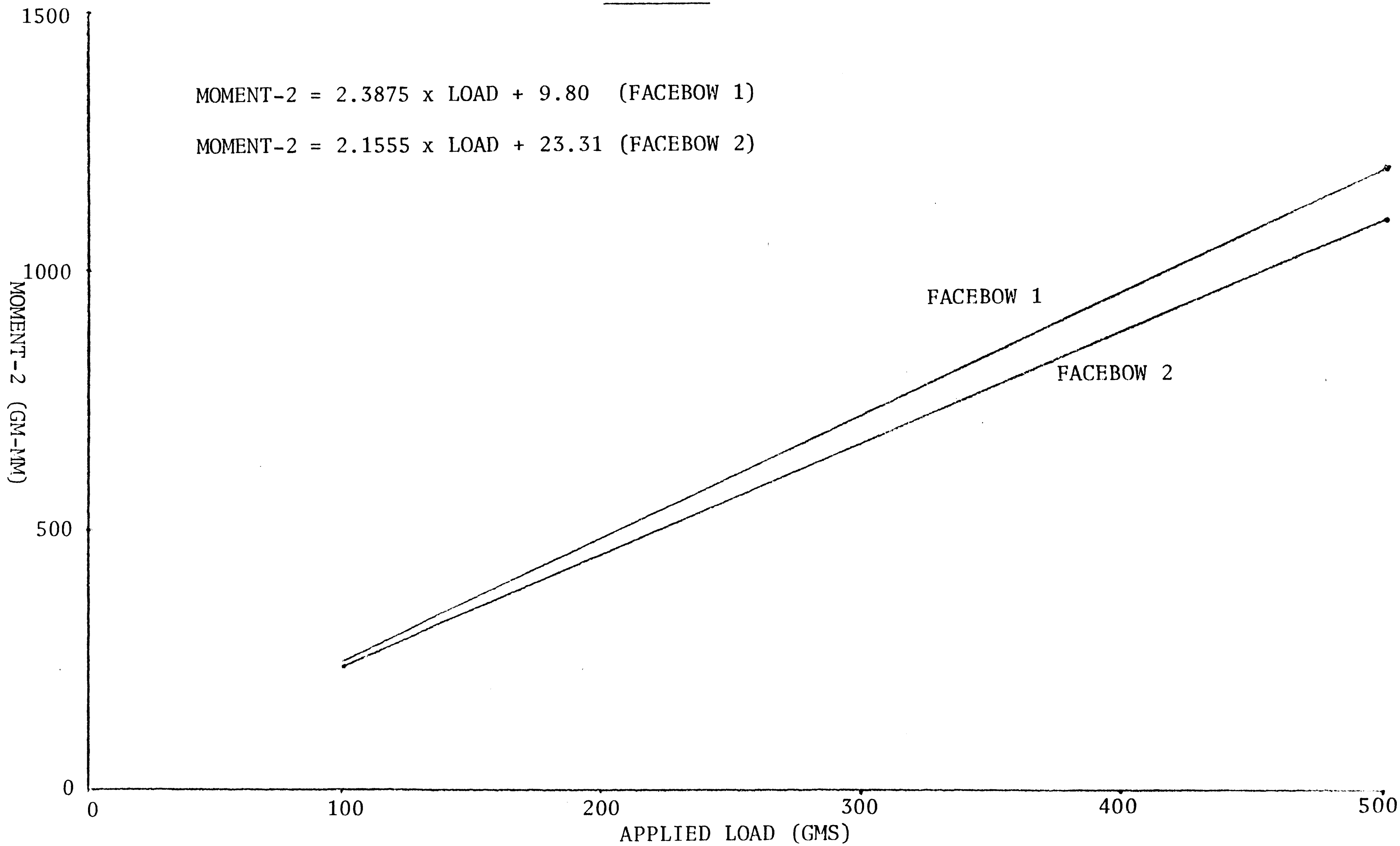
FACEBOW - EXPERIMENTAL - RIGID ATTACHMENT

PLOT II



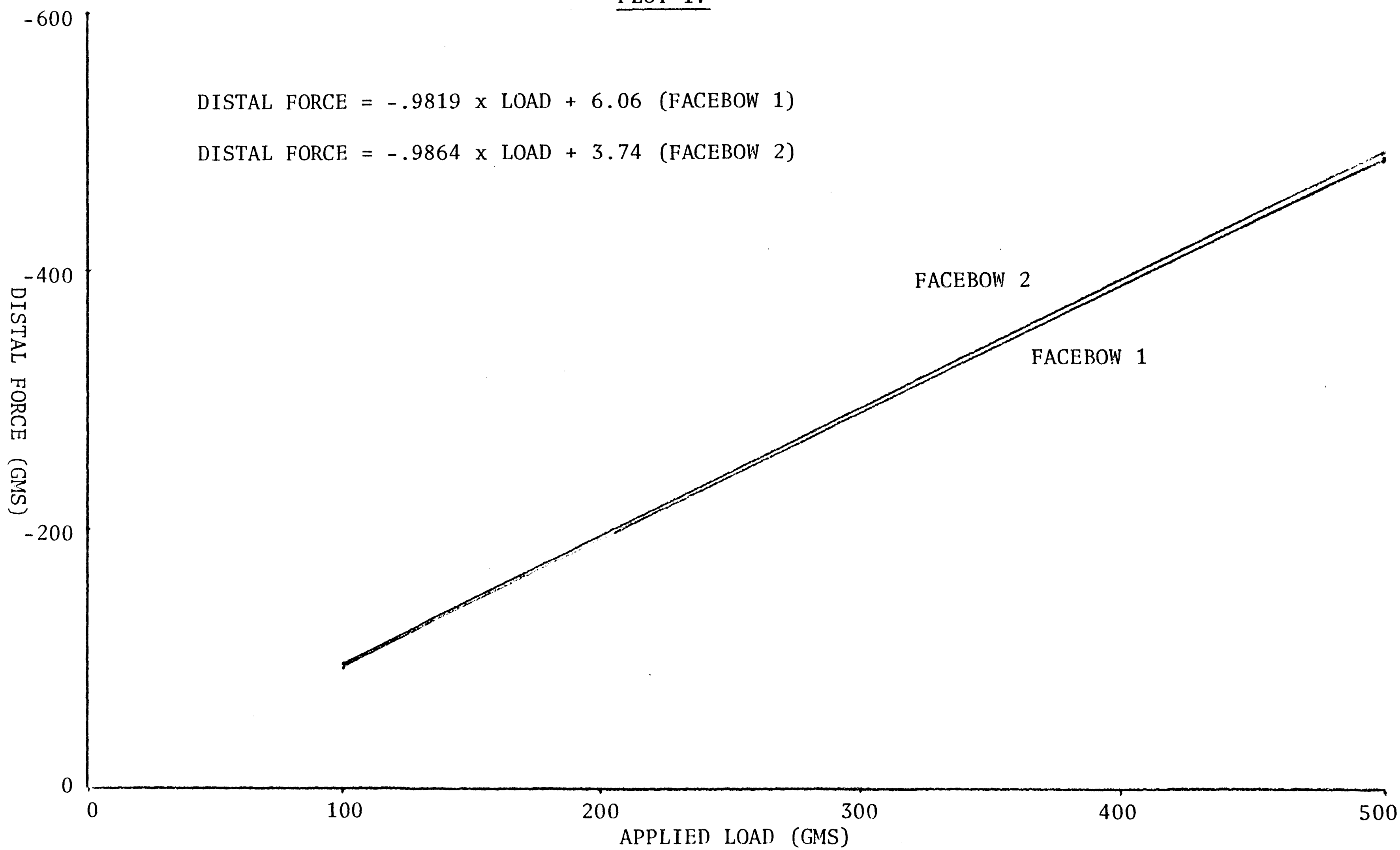
FACEBOW - EXPERIMENTAL - RIGID ATTACHMENT

PLOT III



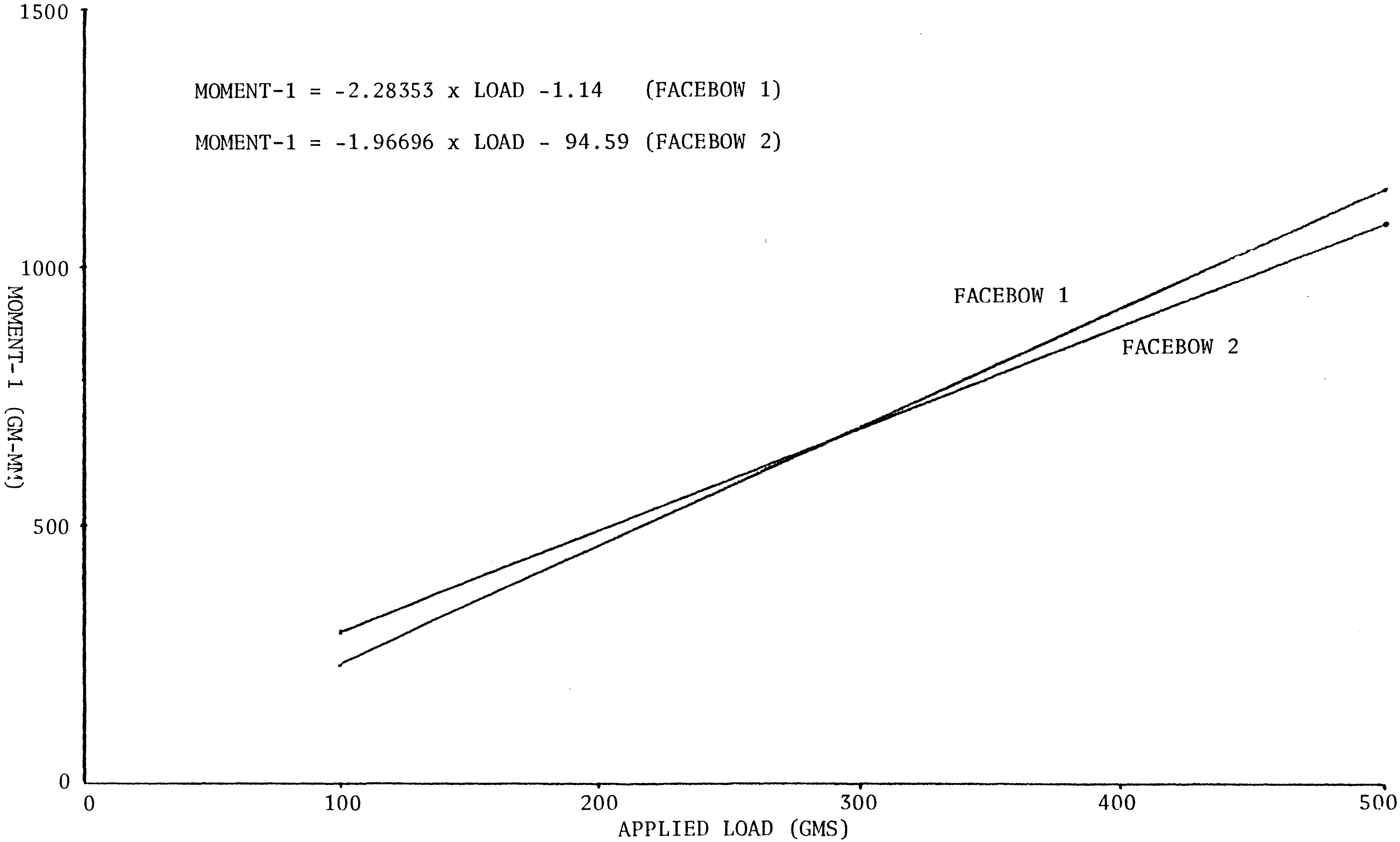
FACEBOW - EXPERIMENTAL - RIGID ATTACHMENT

PLOT IV



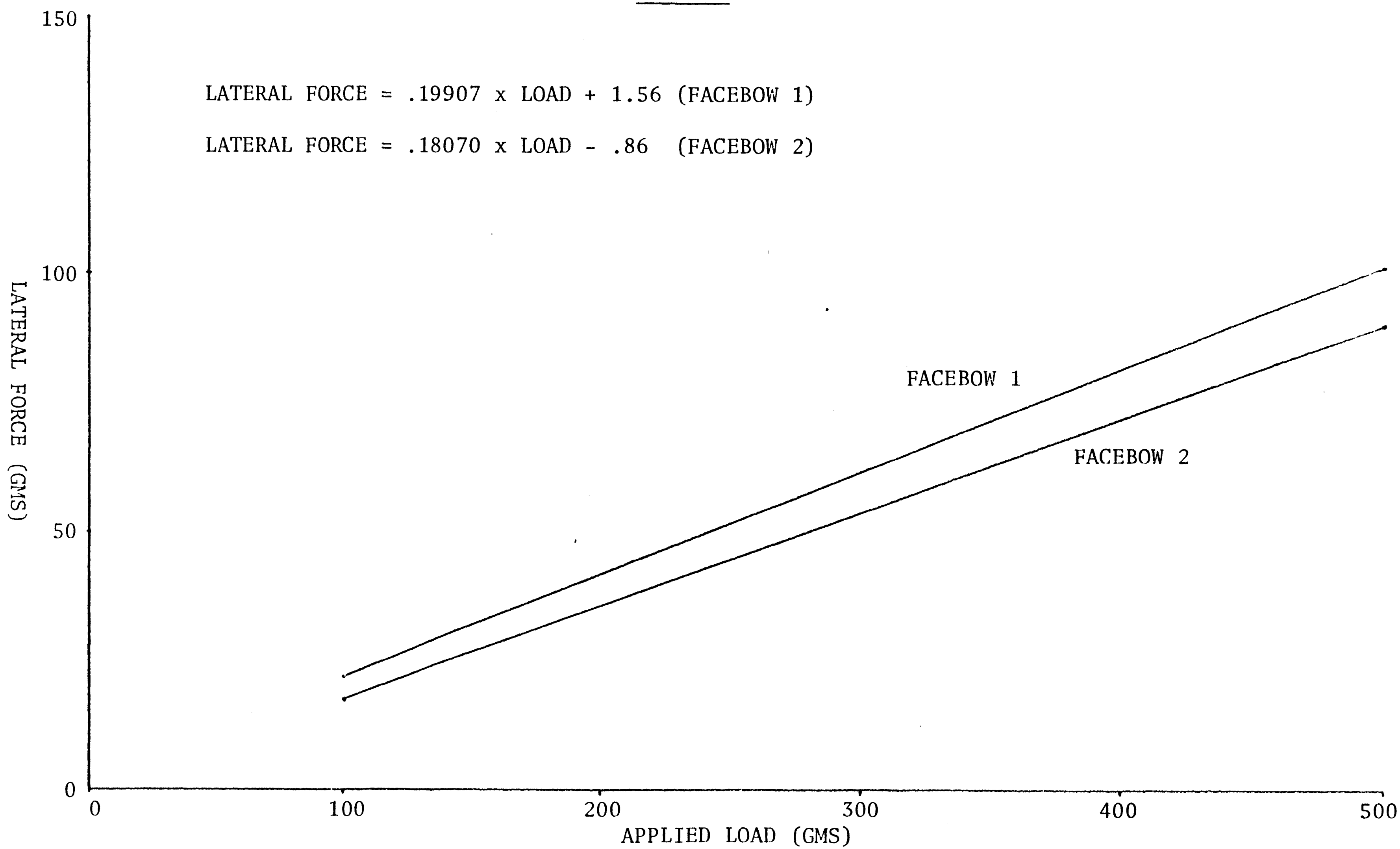
FACEBOW - EXPERIMENTAL - NON-RIGID ATTACHMENT

PLOT V



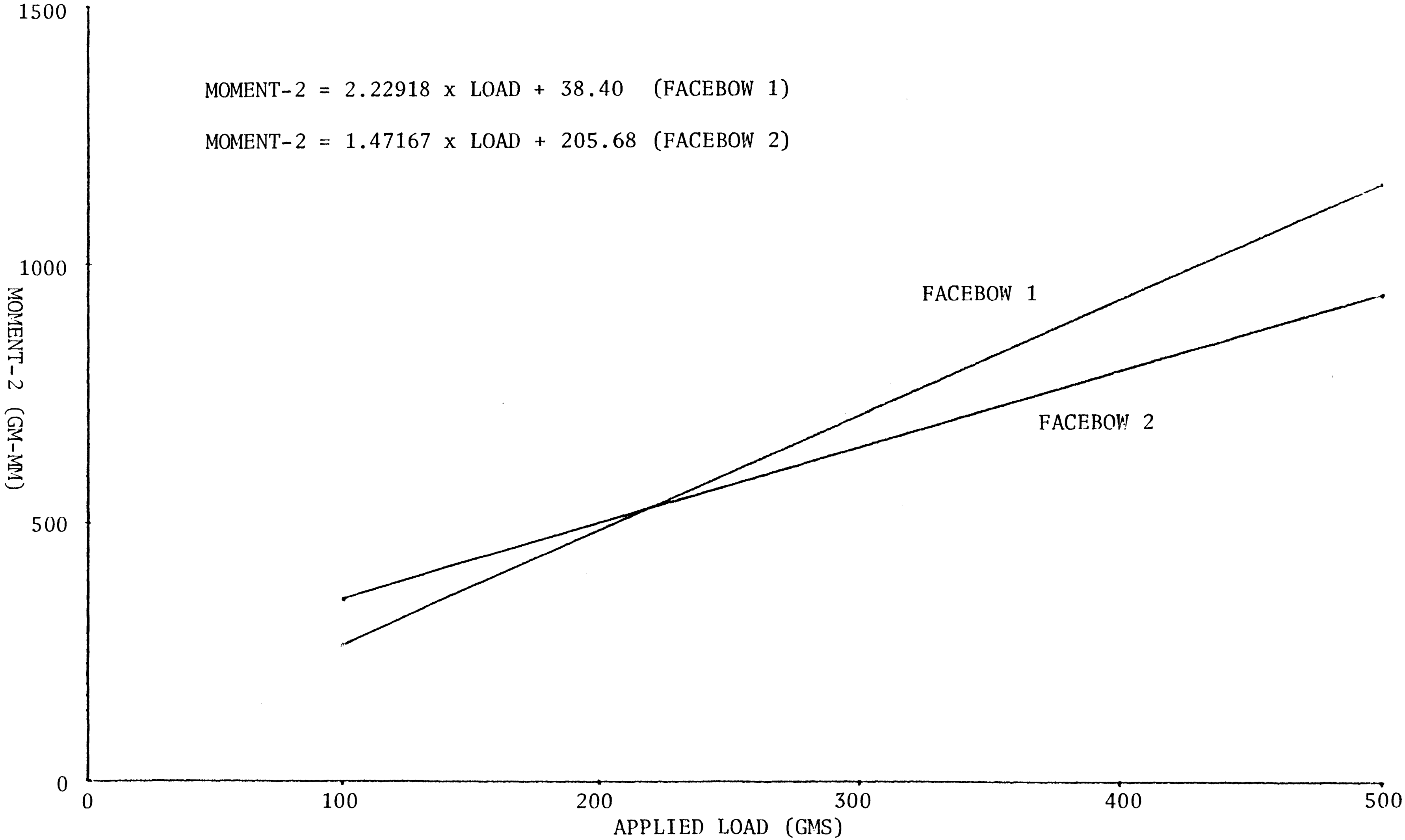
FACEBOW - EXPERIMENTAL - NON-RIGID ATTACHMENT

PLOT VI



FACEBOW - EXPERIMENTAL - NON-RIGID ATTACHMENT

PLOT VII



FACEBOW - EXPERIMENTAL - NON-RIGID ATTACHMENT

PLOT VIII

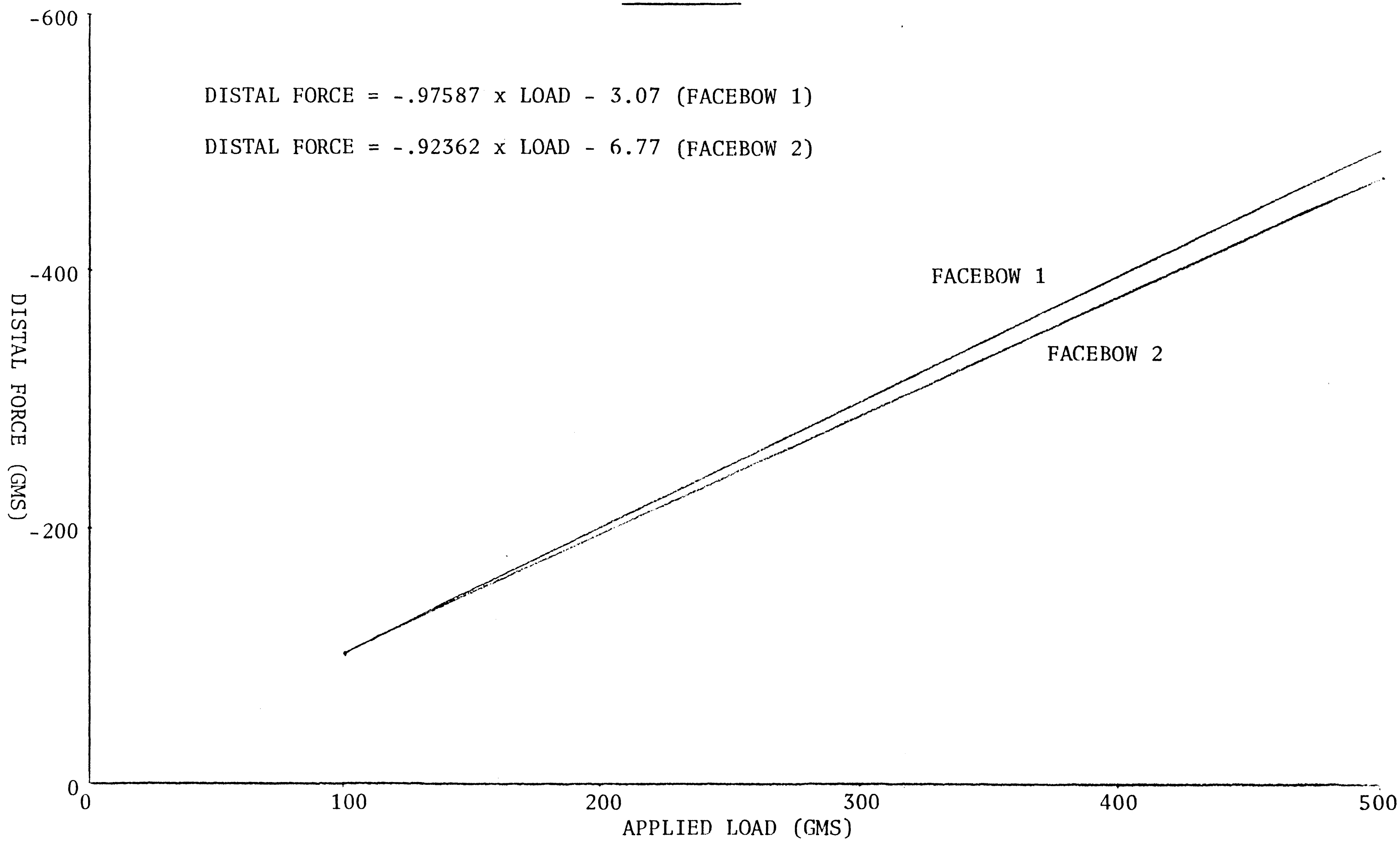


TABLE II
REGRESSION LINE COMPARISON OF FACEBOWS I & II
FOR RIGID AND NON-RIGID ATTACHMENTS

ATTACHMENT	COMPARISON	DEGREES OF FREEDOM	MOMENT-1	LATERAL FORCE	MOMENT-2	DISTAL FORCE
RIGID	ELEVATIONS	1,53	1.553 NS	1.715 NS	.404 NS	.009 NS
	SLOPES	1,51	248.7 SD	228.6 SD	192.3 SD	.39 NS
NON-RIGID	ELEVATIONS	1,98	.001 NS	2.1 NS	1.176 NS	.195 NS
	SLOPES	1,96	23.34 SD	14.97 SD	77.3 SD	97.8 SD

SD = Significant Difference
NS = No Significant Difference

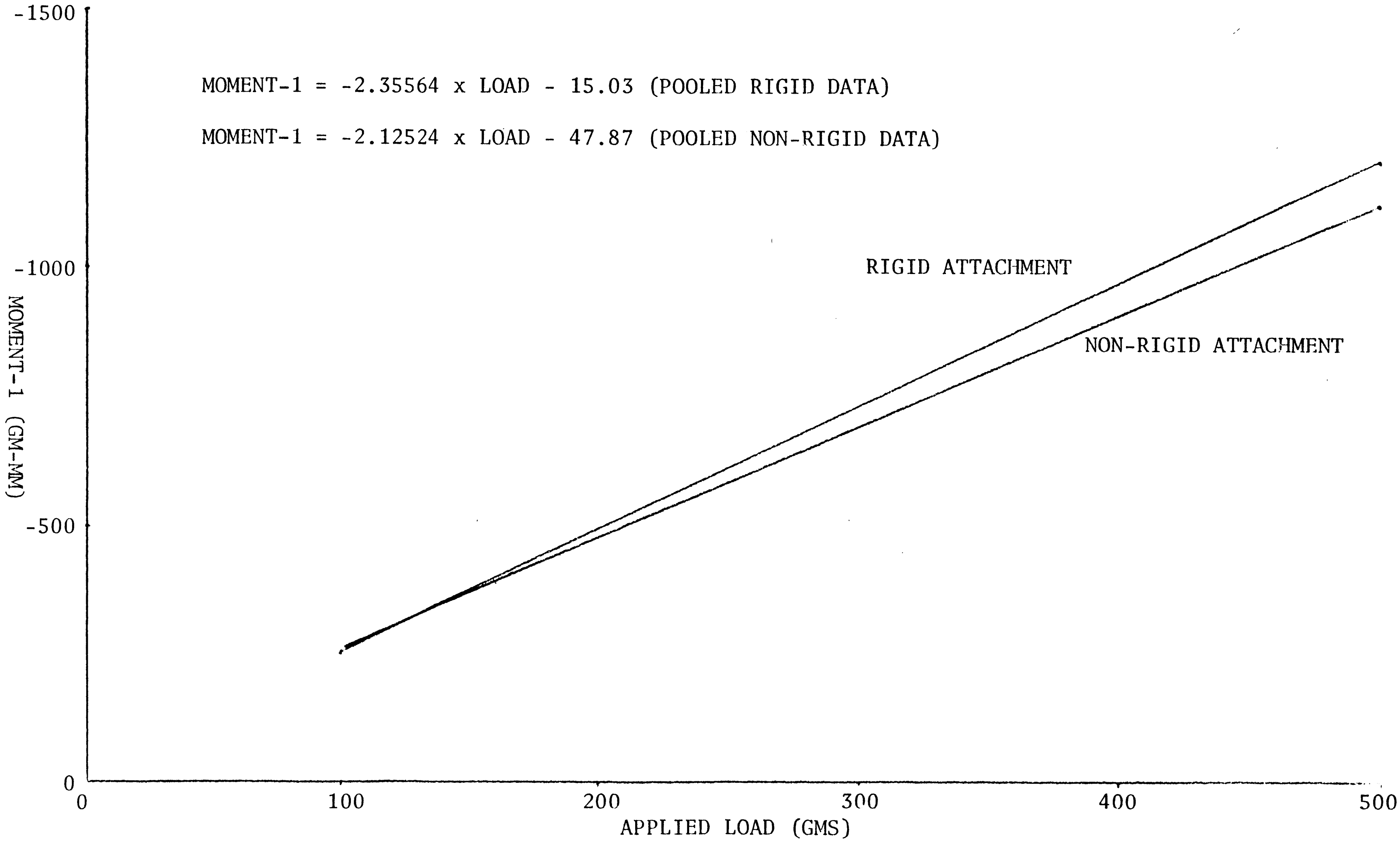
APPENDIX IV

TABLE I
REGRESSION COEFFICIENTS (SLOPES),
INTERCEPTS, & CORRELATION COEFFICIENTS
FOR POOLED FACEBOW DATA TESTED IN RIGID AND NON-RIGID ATTACHMENTS

ATTACHMENT	FACEBOW	STATISTIC	MOMENT-1	LATERAL FORCE	MOMENT-2	DISTAL FORCE
RIGID	POOLED	INTERCEPT:	-15.03	3.48	17.13	4.80
		REGRESSION COEFFICIENT:	-2.35564	.22852	2.26085	-.98438
		CORRELATION COEFFICIENT:	-.9815	.97896	.99456	-.99959
NON-RIGID	POOLED	INTERCEPT:	-47.87	.35	122.04	-4.92
		REGRESSION COEFFICIENT:	-2.12524	.18988	1.85042	-.94974
		CORRELATION COEFFICIENT:	-.98611	.98098	.95044	-.99826

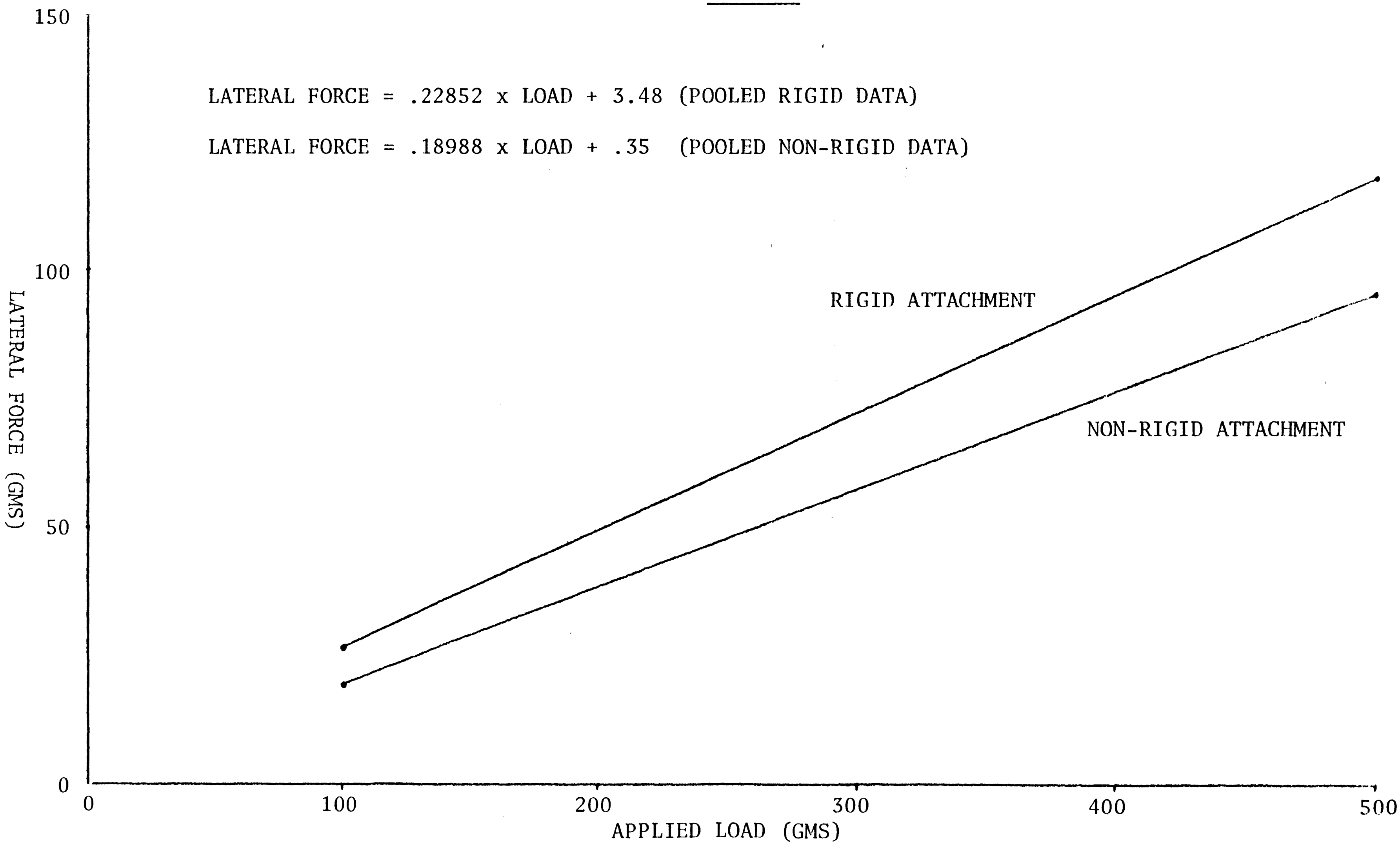
FACEBOW - EXPERIMENTAL - RIGID vs NON-RIGID ATTACHMENT

PLOT I



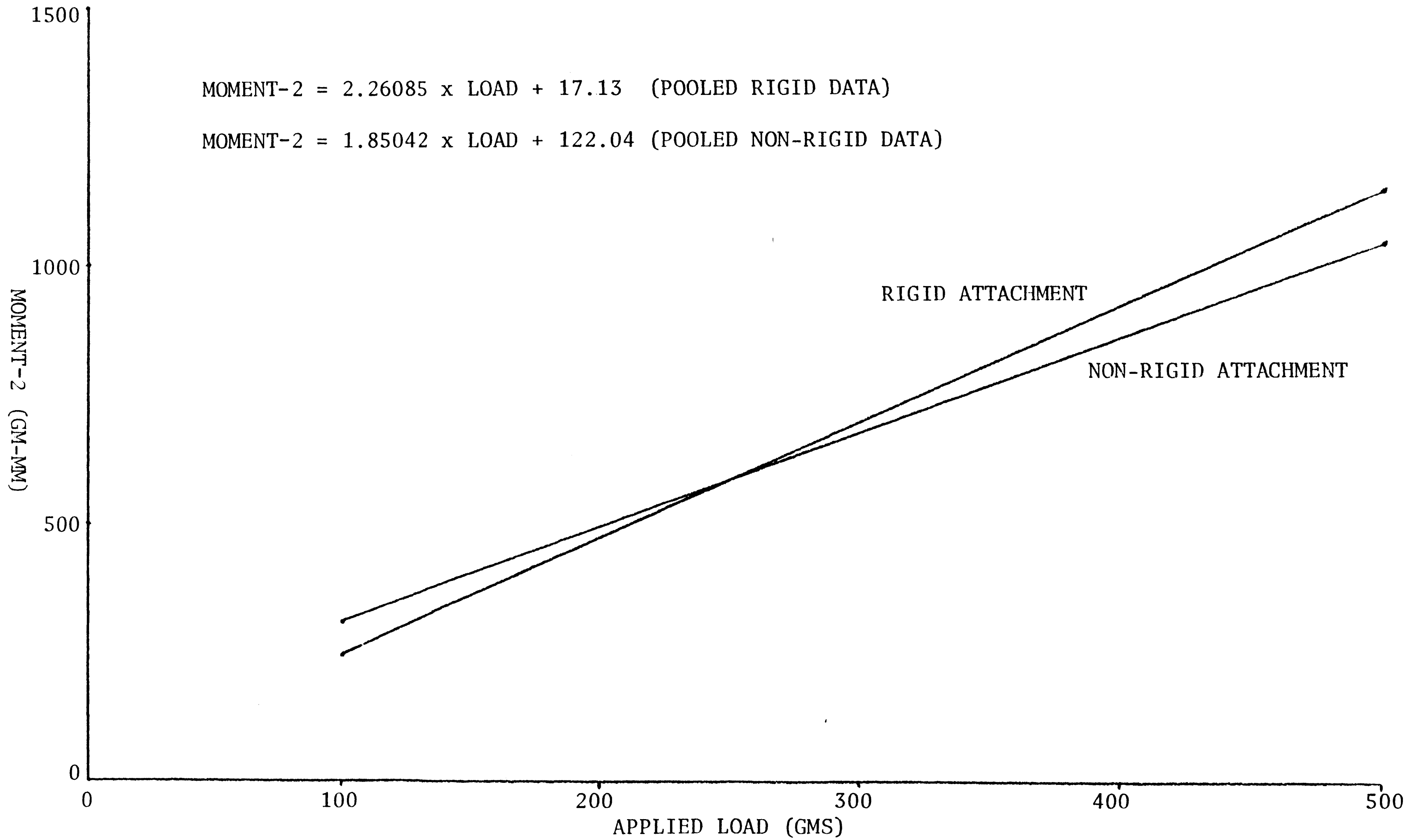
FACEBOW - EXPERIMENTAL - RIGID vs NON-RIGID ATTACHMENT

PLOT II



FACEBOW - EXPERIMENTAL - RIGID vs NON-RIGID ATTACHMENT

PLOT III



FACEBOW - EXPERIMENTAL - RIGID vs NON-RIGID ATTACHMENT

PLOT IV

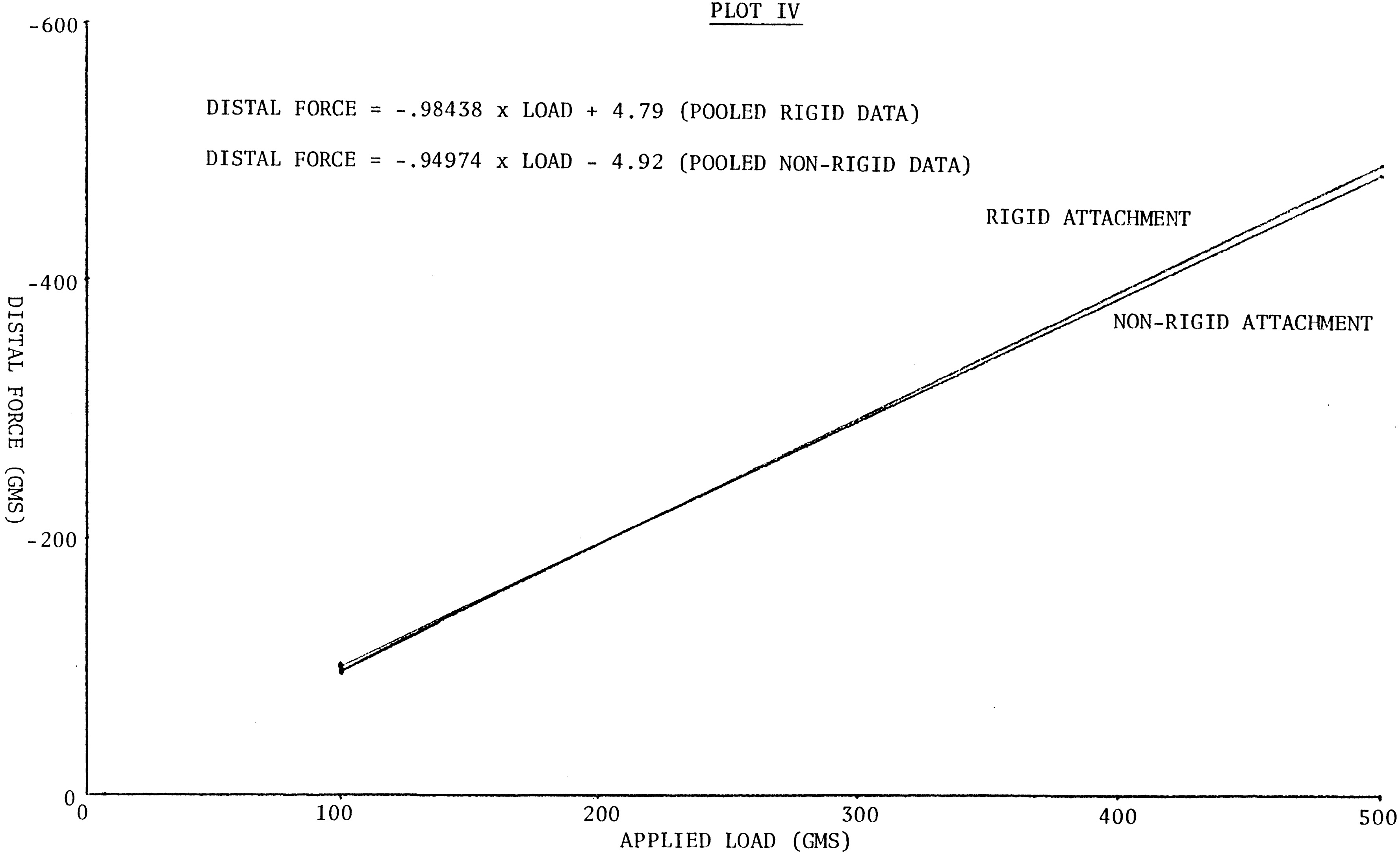


TABLE II

REGRESSION LINE COMPARISON OF RIGID & NON-RIGID ATTACHMENTS

COMPARISON	DEGREES OF FREEDOM	MOMENT-1	LATERAL FORCE	MOMENT-2	DISTAL FORCE
ELEVATIONS	1,153	.457 NS	.136 NS	8.727 SD	.001 NS
SLOPES	1,151	11.646 SD	22.72 SD	30.07 SD	17.913 SD

SD = Significant Difference
NS = No Significant Difference

APPENDIX V

FIGURE A-.045" HEAT TREATMENT SAMPLE

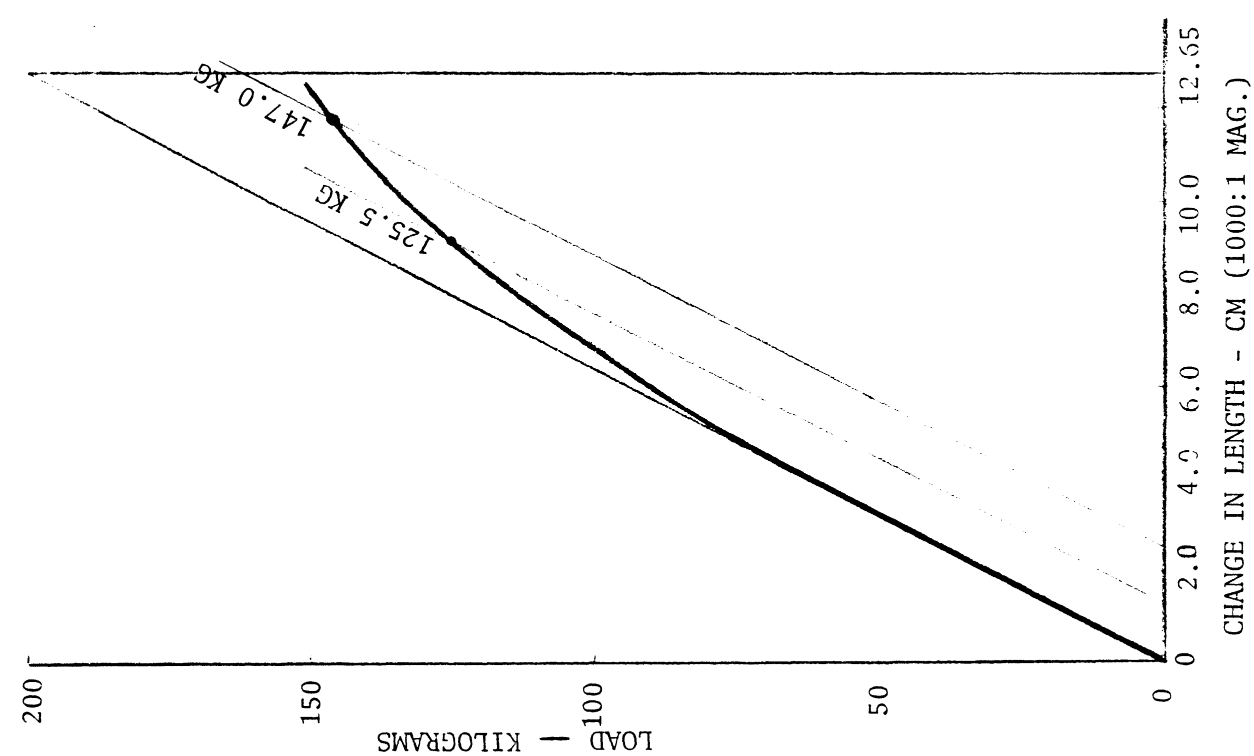
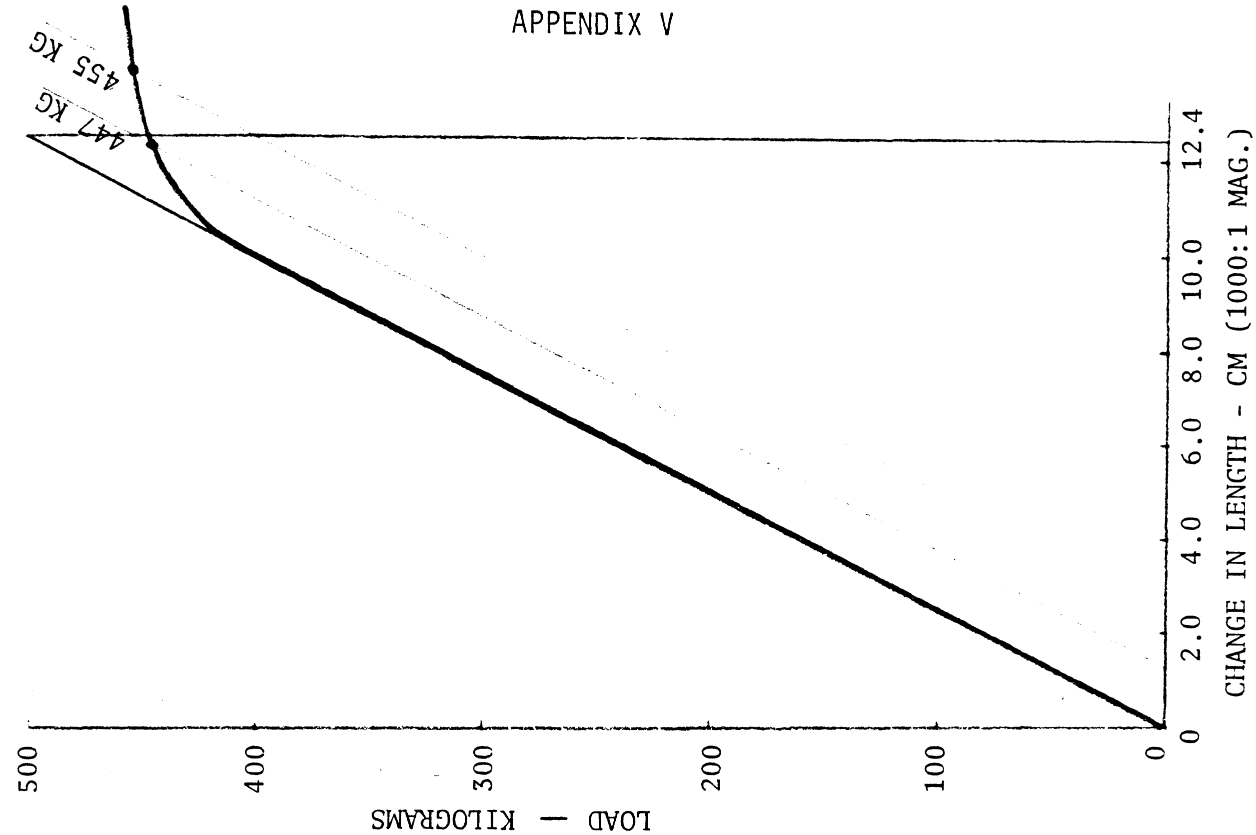


FIGURE B-.072" HEAT TREATMENT SAMPLE



MODULUS OF ELASTICITY AND YIELD STRENGTH CALCULATION

.045" HEAT TREATED

Yield (.1%)

125.5 kg x 2.2 lb/kg = 276.1 lb
Yield = Load/X-Sectional Area
= 276.1 lb/.00159 in^2
= 173,647 psi

Yield (.2%)

147 kg x 2.2 lb/kg = 323.4 lb
Yield = Load/X-Sectional Area
= 323.4 lb/.00159 in^2
= 203,396 psi

Modulus of Elasticity

200 kg x 2.2 lb/kg = 440 lb
Stress = Load/X-Sectional Area
= 440 lb/.00159 in^2
= 276,729 psi

$$\text{Strain} = \frac{L_f - L_o}{L_o} = \frac{\left(1.27 + \frac{12.65}{1000}\right) - 1.27}{1.27} = .0100$$

Modulus = Stress/Strain
= 276,729/.0100
= 27,672,900 psi
= 27.67×10^6 psi

.072" HEAT TREATED

Yield (.1%)

447 kg x 2.2 lb/kg = 983.4 lb
Yield = Load/X-Sectional Area
= 983.4 lb/.00407 in^2
= 241,621 psi

Yield (.2%)

455 kg x 2.2 lb/kg = 1001.0 lb
Yield = Load/X-Sectional Area
= 1001.0 lb/.00407 in^2
= 245,945 psi

Modulus of Elasticity

500 kg x 2.2 lb/kg = 1100 lb

$$\text{Stress} = \text{Load/X-Sectional Area}$$
$$= 1100/.00407 \text{ in}^2$$
$$= 270{,}270 \text{ psi}$$

$$\text{Strain} = \frac{L_f - L_o}{L_o}$$
$$= \frac{\left(1.27 + \frac{12.4}{1000}\right) - 1.27}{1.27}$$
$$= .0097638$$

$$\text{Modulus} = \text{Stress/Strain}$$
$$= 270{,}270/.0097638$$
$$= 27{,}680{,}821 \text{ psi}$$
$$= 27.68 \times 10^6 \text{ psi}$$

TABLE I

MATERIAL PROPERTIES AND ANALYSIS

LOT #!

SAMPLE	MODULUS (x 10^6) psi	YIELD (.1%) psi	YIELD (.2%) psi
.072"	25.06	241,081	245,405
As-Received	28.45	235,135	238,918
	27.80	235,135	238,918
Mean	27.10	228,189	233,513
Standard Deviation	1.80	3,432	3,745
.072"	27.58	229,189	233,513
Heat Treated	29.38	227,027	231,351
Mean	28.48	227,027	231,351
Standard Deviation	1.27	1,528	1,528
.072"	27.46	75,675	97,837
Recrystallization	27.28	80,540	100,000
Mean	27.37	164,654	182,641
Standard Deviation	.13	3,440	1,529
.045"	22.36	164,654	182,641
As-Received	21.80	176,000	196,625
	20.07	191,125	NA
	21.43	199,375	NA
	22.28	193,875	NA
Mean	21.58	185,005	181,500
Standard Deviation	.93	14,300	9,888
.045"	26.40	148,500	181,500
Heat Treated	24.06	155,375	175,312
	24.42	156,750	176,550
Mean	24.96	153,541	177,787
Standard Deviation	1.26	4,420	3,274
.045"	27.94	32,312	32,312
Recrystallization	25.63	34,375	34,375
Mean	26.78	33,343	33,343
Standard Deviation	1.63	1,459	1,459

TABLE II

MATERIAL PROPERTIES AND ANALYSIS

LOT #2

SAMPLE	MODULUS (x 10^6) psi	YIELD (.1%) psi	YIELD (.2%) psi
.072"	26.40	232,432	235,675
As-Received	27.24	237,837	243,243
	29.59	217,297	222,702
	28.02	216,216	220,000
Mean	27.81	225,945	230,405
Standard Deviation	1.36	10,846	10,957
.072"	27.68	235,135	239,459
Heat-Treated	27.24	232,972	237,297
	27.68	241,621	245,945
	28.13	243,243	247,027
Mean	27.68	238,242	242,432
Standard Deviation	.36	4,961	4,784
.045"	24.27	186,792	208,930
As-Received	23.86	188,176	215,849
	24.40	NA	NA
	25.79	185,408	207,547
	22.14	200,628	220,000
Mean	24.09	190,251	213,081
Standard Deviation	1.31	7,009	5,870
.045"	30.41	167,421	196,477
Heat Treated	27.67	173,647	203,396
	21.96	196,477	215,849
	24.49	188,176	210,314
Mean	26.13	181,430	206,509
Standard Deviation	3.69	13,275	8,407

APPENDIX VI

TABLE I

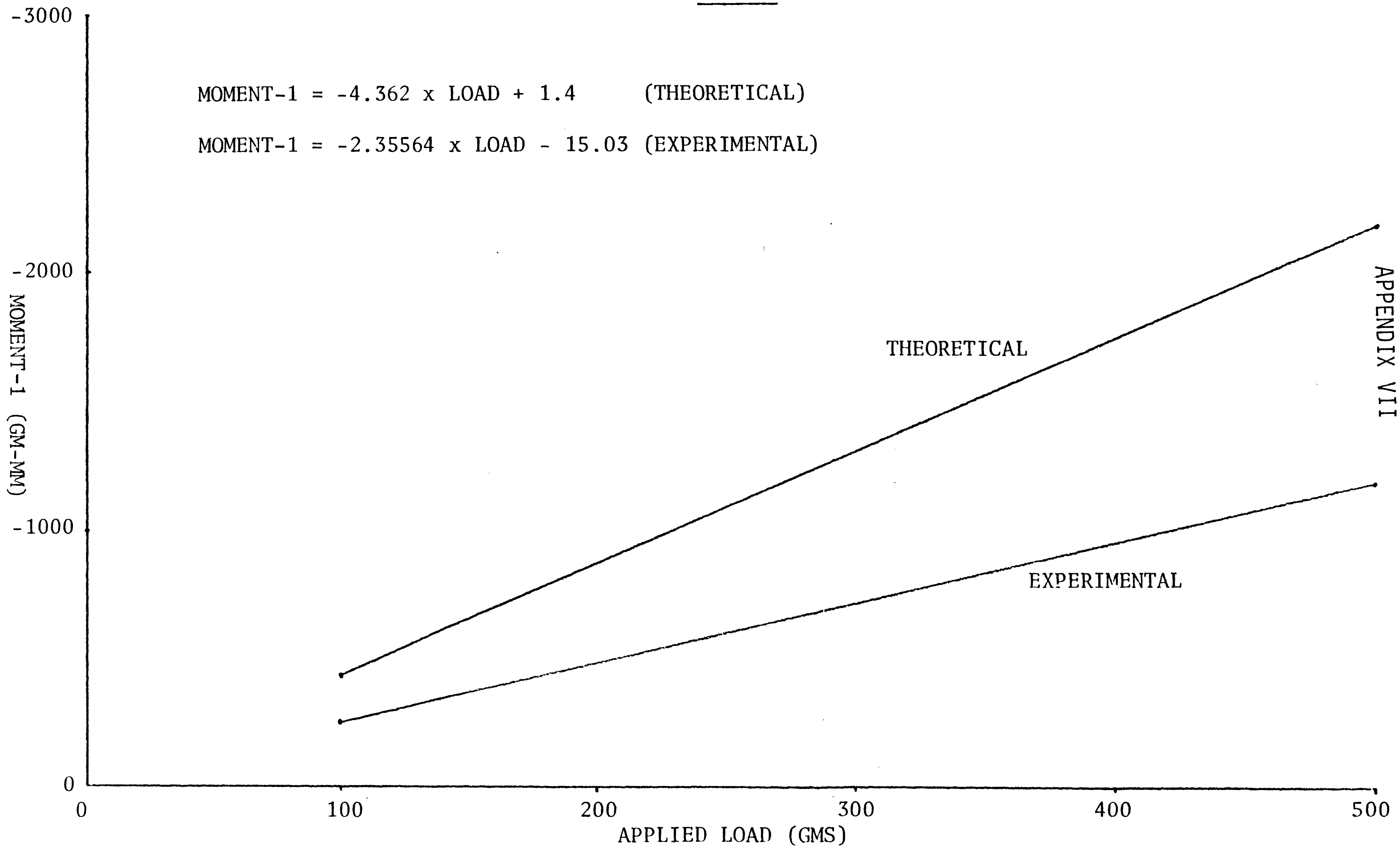
FACEBOW - RIGID ATTACHMENT

THEORETICAL FORCE SYSTEM VALUES

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms) (1)	DISTAL FORCE (gms) (2)
100	-435.0	36.4	363.0	-97.1	-100.0
200	-871.0	72.8	727.0	-194.2	-200.5
300	-1307.0	109.2	1091.0	-291.3	-301.0
400	-1743.0	145.6	1455.0	-388.4	-401.5
500	-2180.0	182.0	1820.0	-486.0	-502.0

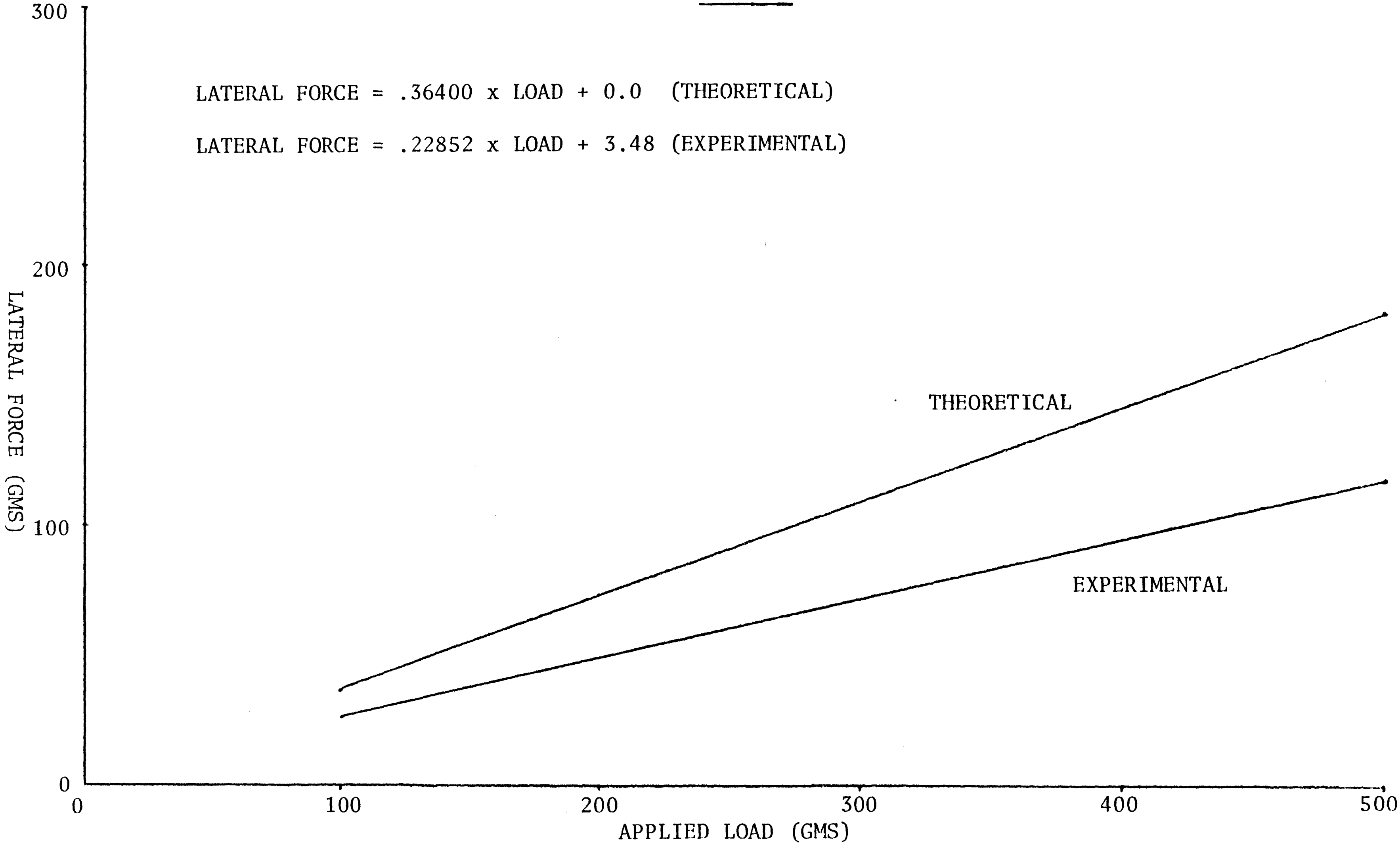
FACEBOW - RIGID ATTACHMENT - THEORETICAL vs EXPERIMENTAL

PLOT I



FACEBOW - RIGID ATTACHMENT - THEORETICAL vs EXPERIMENTAL

PLOT II

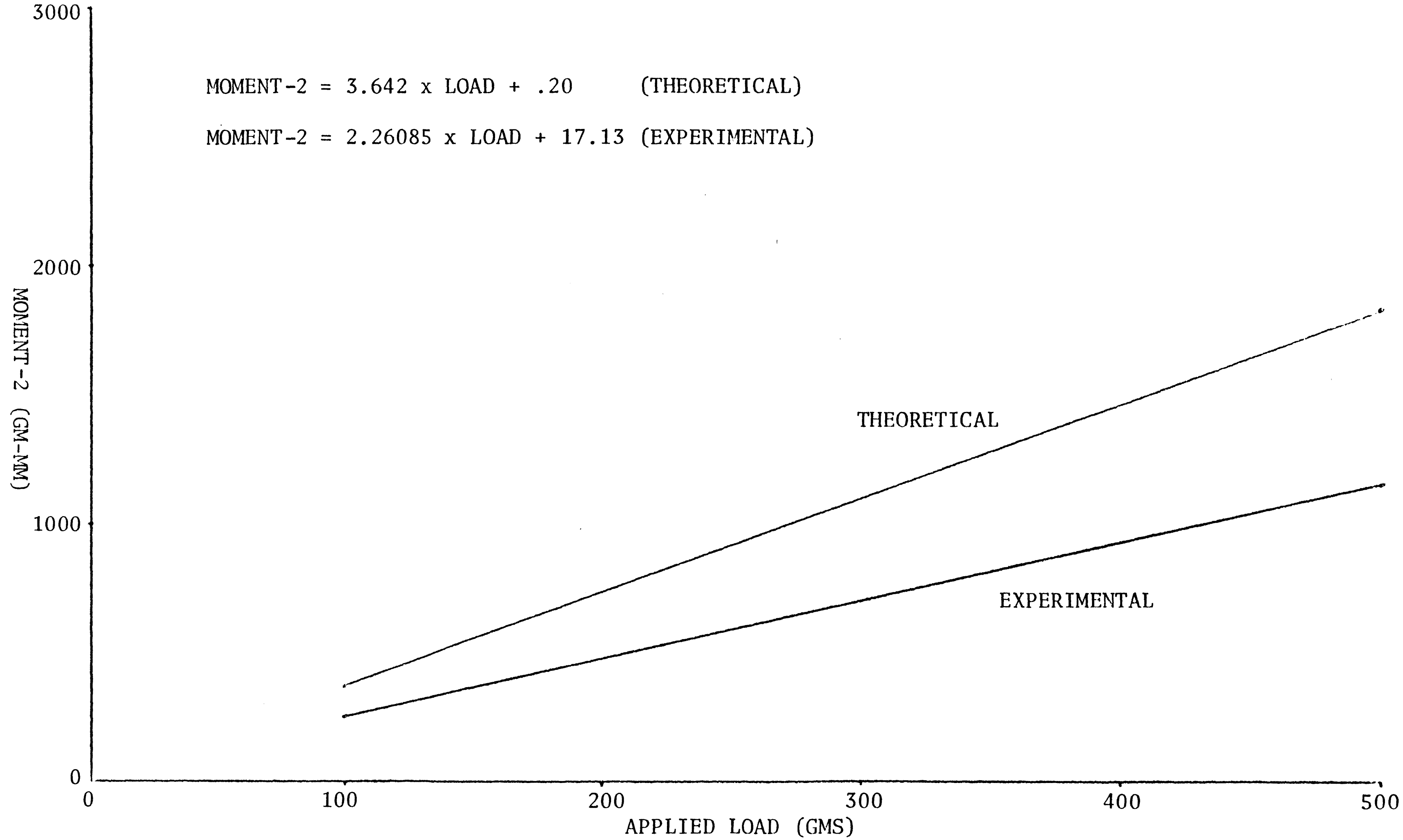


FACEBOW - RIGID ATTACHMENT - THEORETICAL vs EXPERIMENTAL

PLOT III

MOMENT-2 = 3.642 x LOAD + .20 (THEORETICAL)

MOMENT-2 = 2.26085 x LOAD + 17.13 (EXPERIMENTAL)



FACEBOW - RIGID ATTACHMENT - THEORETICAL vs EXPERIMENTAL

PLOT IV

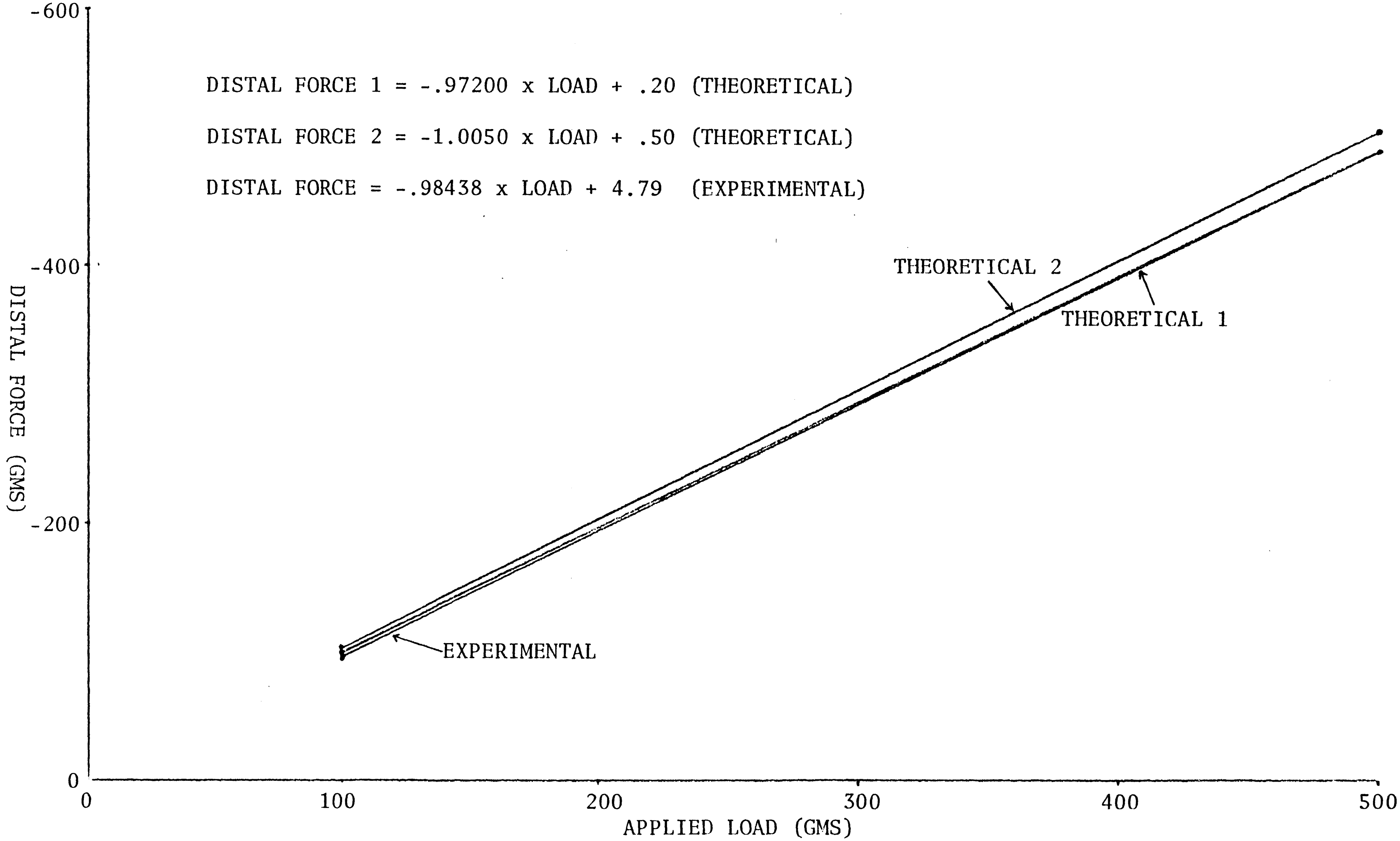


TABLE 1

THEORETICAL ANALYSIS OF THE
EFFECT OF ATTACHMENT ROTATION ON FORCE SYSTEM

ROTATION (degrees)	MOMENT-1	LATERAL FORCE	MOMENT-2	DISTAL FORCE (1)	DISTAL FORCE (2)
0	-2180	+182	+1820	-486	-502
.5	-776	+111	+404	-485	-502
1.0	+623	+40.6	-1010	-485	-503

EFFECT OF BRACKET ROTATION ON FORCE SYSTEM

PLOT V

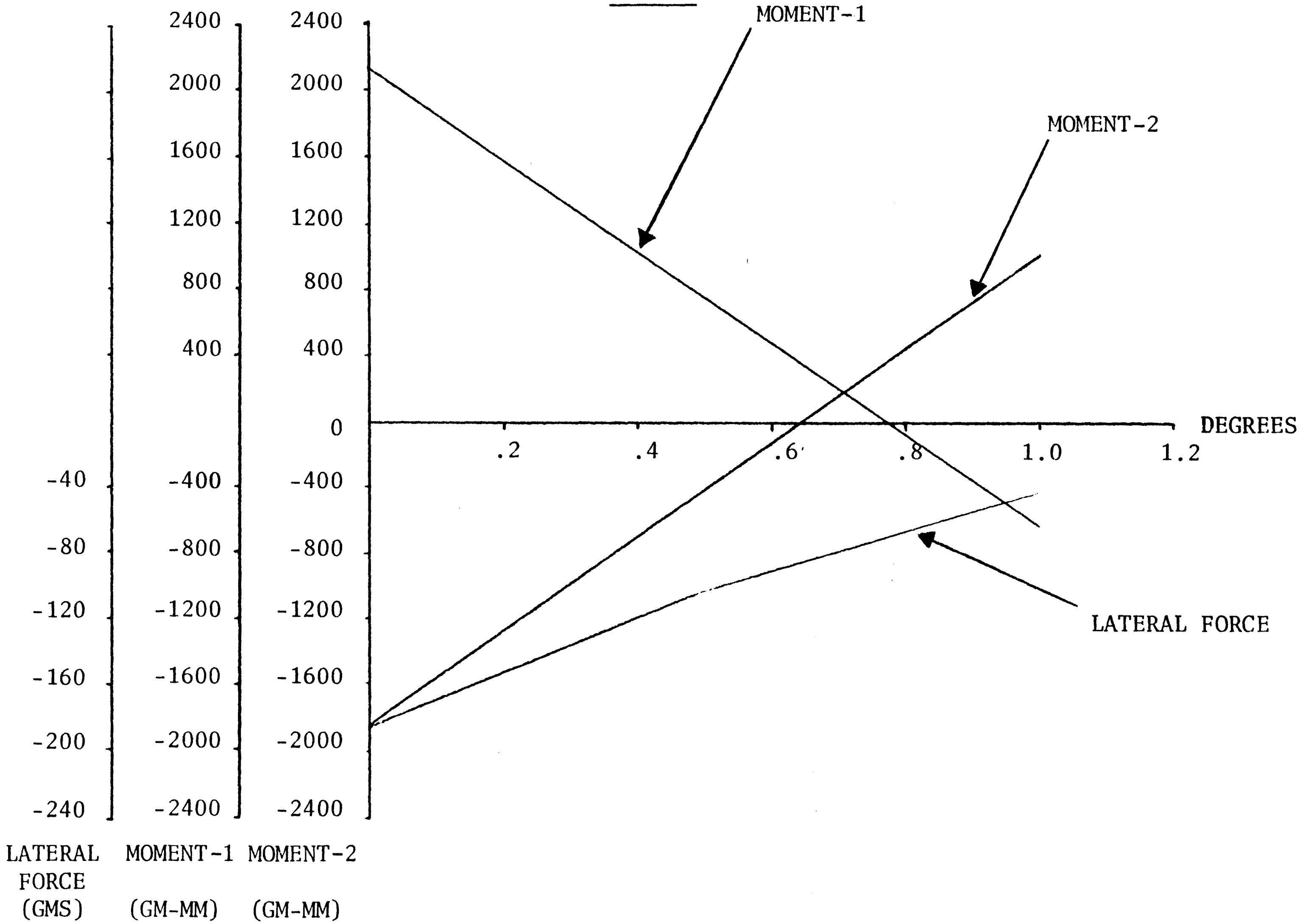


TABLE II

THEORETICAL ANALYSIS OF THE
EFFECT IF ATTACHMENT POSITION ON FORCE SYSTEM

ATTACHMENT	MOMENT-1	LATERAL FORCE	MOMENT-2	DISTAL FORCE	
				(1)	(2)
AT MOLAR OFFSET	-2180	182	1820	-486	-502
7 MM DISTAL TO OFFSET	-1480	126	1340	-485	-503

APPENDIX VIII

TABLE I

DIVERGENT ARCH - RIGID ATTACHMENT

5 SAMPLES

UNCORRECTED FORCE SYSTEM VALUES

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
100	-122.56	22.38	125.59	-47.12
200	-234.09	44.37	250.44	-95.04
300	-337.78	65.99	355.63	-140.68
400	-416.22	87.79	462.22	-191.71
500	-519.90	111.23	578.62	-244.66
600	-676.91	132.09	653.11	-287.41
700	-753.27	154.00	792.49	-343.93
800	-901.07	179.75	894.73	-393.19
900	-1049.74	199.06	982.89	-436.06
1000	-1164.70	225.50	1130.29	-494.05
100	-124.88	26.29	123.63	-47.04
200	-231.88	45.2	248.72	-94.38
300	-332.26	66.57	365.43	-143.33
400	-421.99	88.23	466.76	-193.26
500	-494.53	110.61	575.19	-241.94
600	-628.50	131.91	676.97	-291.22
700	-770.42	152.36	775.70	-338.32
800	-881.21	176.89	912.56	-393.45
900	-1056.24	204.52	1005.62	-439.80
1000	-1182.84	224.52	1115.47	-483.26
100	-125.74	24.16	131.96	-50.33
200	-237.40	45.42	259.14	-99.76
300	-322.83	68.74	361.75	-147.62
400	-433.87	86.13	454.32	-192.89
500	-509.37	109.09	581.50	-245.81
600	-628.74	128.98	663.28	-289.59
700	-777.53	152.55	752.55	-337.22
800	-881.95	176.39	896.88	-395.85
900	-1005.74	199.06	1000.29	-443.55
1000	-1120.95	221.12	1122.82	-495.74

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
100	-133.59	24.34	127.73	-50.22
200	-250.27	47.30	256.14	-99.36
300	-344.03	66.61	373.39	-145.28
400	-418.80	89.21	467.06	-193.43
500	-519.54	110.76	564.78	-240.97
600	-620.00	131.80	671.19	-292.82
700	-753.27	155.67	777.84	-340.60
800	-855.36	177.62	911.03	-396.81
900	-1015.54	197.21	1008.80	-444.92
1000	-1153.80	223.18	1117.00	-493.90
100	-125.63	24.30	129.38	-49.01
200	-236.17	47.45	251.98	-95.83
300	-344.27	67.55	352.25	-141.20
400	-439.38	87.50	456.83	-191.51
500	-529.96	110.39	561.22	-241.00
600	-653.75	133.79	658.57	-289.43
700	-794.81	153.89	757.51	-335.18
800	-920.41	177.18	875.68	-385.63
900	-1025.35	198.01	994.29	-437.39
1000	-1165.07	222.20	1101.67	-489.10

TABLE II

DIVERGENT ARCH INVERTED - RIGID ATTACHMENT

4 SAMPLES

UNCORRECTED FORCE SYSTEM VALUES

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
100	-125.58	22.80	131.38	-46.86
200	-246.08	48.74	260.90	-94.80
300	-355.52	66.67	391.41	-138.23
400	-442.93	91.66	467.29	-187.75
500	-542.23	111.42	558.06	-235.39
600	-661.09	133.49	651.25	-280.80
700	-787.14	153.94	747.97	-328.62
800	-917.78	178.31	854.48	-376.85
900	-1058.79	200.63	954.30	-427.17
1000	-1189.78	225.33	1054.68	-476.81
100	-128.87	23.86	136.09	-46.48
200	-247.03	44.46	260.03	-92.91
300	-341.15	67.51	387.13	-141.55
400	-416.19	89.28	484.96	-190.29
500	-506.54	114.24	592.41	-238.08
600	-614.56	132.98	694.90	-288.34
700	-745.20	156.06	787.34	-333.61
800	-863.00	179.15	905.94	-384.63
900	-982.10	202.72	993.43	-427.52
1000	-1125.70	225.96	1122.32	-483.95
100	-125.81	25.14	130.14	-46.50
200	-245.85	46.65	254.20	-91.49
300	-346.57	68.87	378.57	-138.85
400	-427.14	90.60	477.34	-186.65
500	-526.80	110.73	574.62	-235.35
600	-640.71	134.15	669.85	-282.53
700	-759.69	155.30	775.00	-332.58
800	-867.95	180.73	892.80	-382.78
900	-1011.55	203.45	1001.11	-430.61
1000	-1138.77	225.96	1115.50	-480.06

LOAD	MOMENT-1	LATERAL FORCE	MOMENT-2	DISTAL FORCE
(gms)	(gm-mm)	(gms)	(gm-mm)	(gms)
100	-125.1	25.36	131.13	-45.46
200	-241.81	46.25	270.26	-93.97
300	-322.42	69.85	390.85	-142.55
400	-434.64	90.42	465.25	-184.41
500	-533.28	112.56	564.45	-233.63
600	-638.71	133.45	657.20	-278.58
700	-768.64	156.25	756.59	-325.74
800	-901.52	178.13	850.58	-372.13
900	-1041.94	202.32	954.55	-419.98
1000	-1175.88	225.59	1066.52	-470.44

APPENDIX IX

TABLE I

DIVERGENT ARCH - RIGID ATTACHMENT

5 SAMPLES

CORRECTED FORCE SYSTEM VALUES

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
100	-143.6	24.7	161.6	-47.2
	-145.9	28.6	159.6	-47.1
	-145.0	26.4	168.0	-50.4
	-153.6	26.6	163.7	-50.3
	-145.5	26.6	165.4	-49.1
200	-282.1	49.0	334.4	-95.0
	-279.9	49.8	332.7	-94.4
	-285.4	50.0	348.1	-99.8
	-298.3	51.9	345.1	-99.4
	-284.2	52.0	336.0	-95.8
300	-414.8	72.3	497.6	-140.7
	-411.3	72.9	508.4	-143.3
	-400.8	75.0	506.7	-147.6
	-423.0	72.9	515.4	-145.3
	-421.3	74.1	494.2	-141.2
400	-499.2	96.1	665.2	-191.7
	-505.0	96.5	669.8	-193.3
	-516.9	94.4	659.3	-192.9
	-501.8	97.5	670.1	-193.4
	-522.4	95.8	661.8	-191.5
500	-670.0	122.4	864.6	-245.7
	-647.0	121.8	857.2	-242.8
	-660.0	120.2	867.5	-246.8
	-672.5	121.8	847.7	-241.9
	-682.9	121.4	845.2	-242.0
600	-841.0	144.3	1060.0	-287.2
	-793.5	143.6	1081.0	-290.8
	-793.7	140.6	1066.2	-289.2
	-785.0	143.4	1074.2	-292.4
	-818.7	146.0	1065.6	-289.0

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
700	-1023.3	174.4	1260.5	-343.8
	-1037.4	172.8	1247.7	-338.2
	-1047.5	172.9	1224.6	-337.1
	-1026.3	176.1	1245.8	-340.5
	-1064.8	174.3	1231.5	-335.1
800	-1257.1	196.9	1468.7	-393.1
	-1237.2	194.1	1484.6	-393.3
	-1237.9	193.6	1468.9	-395.7
	-1211.4	194.8	1479.0	-396.7
	-1276.4	194.4	1451.7	-385.5
900	-1468.7	218.2	1641.9	-435.9
	-1475.2	221.6	1666.6	-439.6
	-1424.7	218.2	1661.3	-443.3
	-1434.5	216.3	1669.8	-444.7
	-1444.0	217.1	1653.3	-437.2
1000	-1801.7	246.2	1890.3	-493.3
	-1819.8	245.2	1877.5	-482.5
	-1757.9	241.8	1878.8	-494.9
	-1790.8	243.9	1877.0	-493.1
	-1798.1	242.9	1861.7	-488.3

TABLE II

DIVERGENT ARCH INVERTED - RIGID ATTACHMENT

4 SAMPLES

CORRECTED FORCE SYSTEM VALUES

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
100	-147.6	26.4	162.0	-46.9
	-150.9	27.5	167.1	-46.5
	-147.8	28.7	161.1	-46.5
	-147.1	29.0	163.1	-45.5
200	-305.1	55.7	345.9	-95.0
	-306.0	51.5	345.0	-93.1
	-304.8	53.6	339.2	-91.7
	-300.6	53.2	355.3	-94.2
300	-451.5	77.0	537.4	-138.4
	-437.0	77.8	531.1	-141.8
	-442.6	79.2	522.6	-139.0
	-418.4	80.1	530.8	-142.7
400	-582.9	104.3	686.3	-187.9
	-556.2	101.9	701.0	-190.5
	-567.1	103.2	694.3	-186.6
	-574.0	103.0	683.2	-184.6
500	-753.2	130.7	875.1	-235.4
	-720.5	133.5	906.4	-238.1
	-738.8	130.0	890.6	-235.3
	-744.3	131.9	881.4	-233.6
600	-895.1	151.3	1025.2	-280.8
	-853.6	150.8	1062.9	-288.3
	-876.6	151.9	1039.8	-282.5
	-872.7	151.2	1027.2	-278.6
700	-1079.0	174.6	1217.0	-328.6
	-1042.2	176.7	1252.3	-333.6
	-1055.7	176.0	1241.0	-332.6
	-1059.6	176.9	1223.0	-325.7
800	-1280.8	201.8	1424.5	-376.8
	-1240.0	203.2	1471.9	-384.6
	-1244.9	204.8	1458.8	-382.8
	-1260.5	201.6	1415.6	-372.1

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
900	-1582.8	226.8	1569.3	-427.2
	-1508.1	228.9	1610.4	-427.5
	-1537.5	229.6	1620.1	-430.6
	-1566.9	228.5	1571.0	-420.0
1000	-1670.8	253.9	1813.7	-476.8
	-1629.7	254.0	1877.3	-483.9
	-1633.8	254.2	1869.5	-480.1
	-1651.9	254.0	1818.5	-470.4

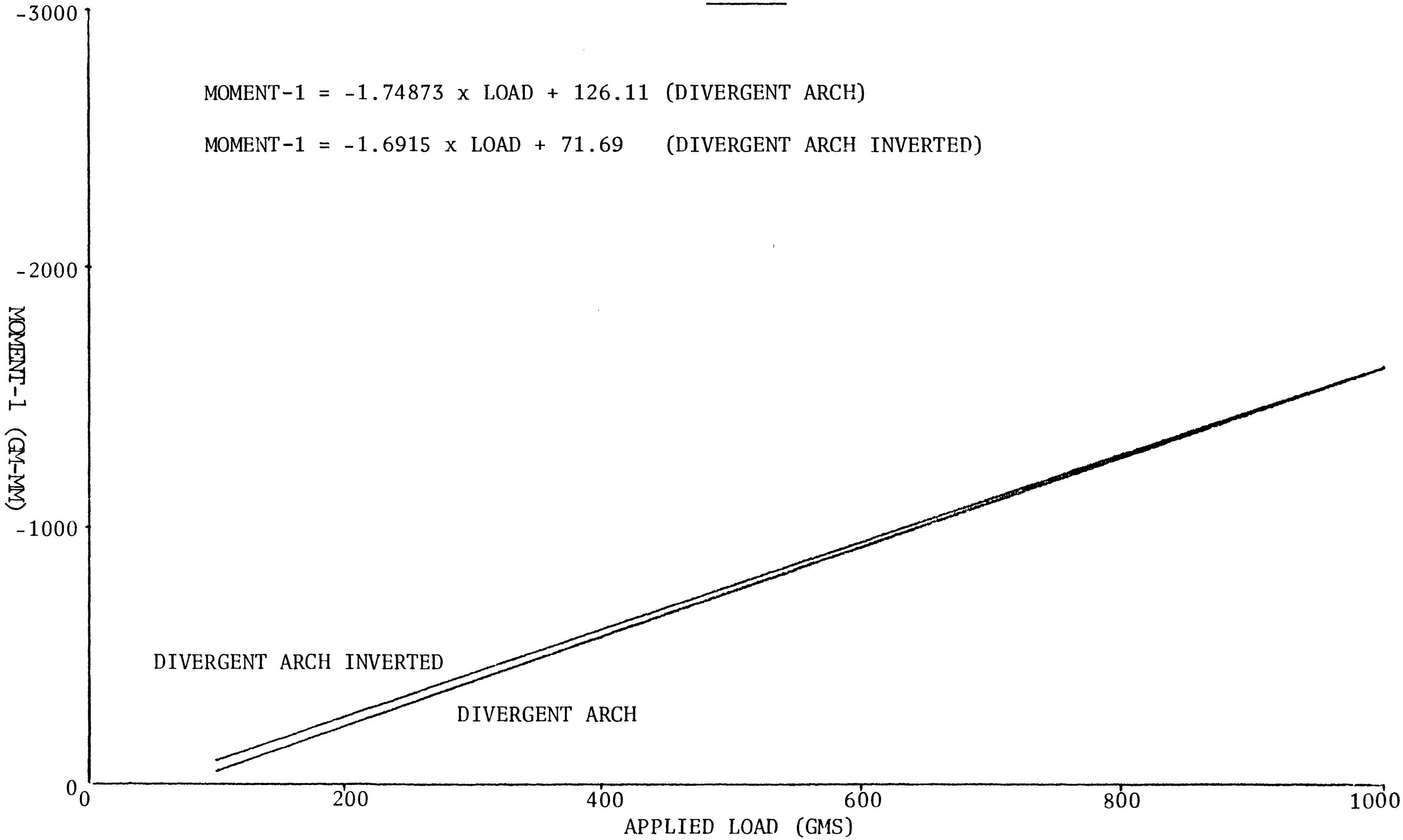
APPENDIX X

TABLE I
REGRESSION COEFFICIENTS (SLOPES), INTERCEPTS, & CORRELATION COEFFICIENTS FOR THE DIVERGENT ARCH AND ITS INVERSION TESTED IN THE RIGID ATTACHMENT

ATTACHMENT	MODEL	STATISTIC	MOMENT-1	LATERAL FORCE	MOMENT-2	DISTAL FORCE
RIGID	DIVERGENT ARCH	INTERCEPT:	126.11	1.23	-63.25	2.51
		REGRESSION COEFFICIENT:	-1.74873	.24185	1.90558	-.49146
		CORRELATION COEFFICIENT:	-.98558	.99953	.99881	-.99973
RIGID	DIVERGENT ARCH INVERTED	INTERCEPT:	71.69	3.34	-29.64	2.87
		REGRESSION COEFFICIENT:	-1.6915	.24983	1.83097	-.47789
		CORRELATION COEFFICIENT:	-.99429	.99969	.99873	-.99968

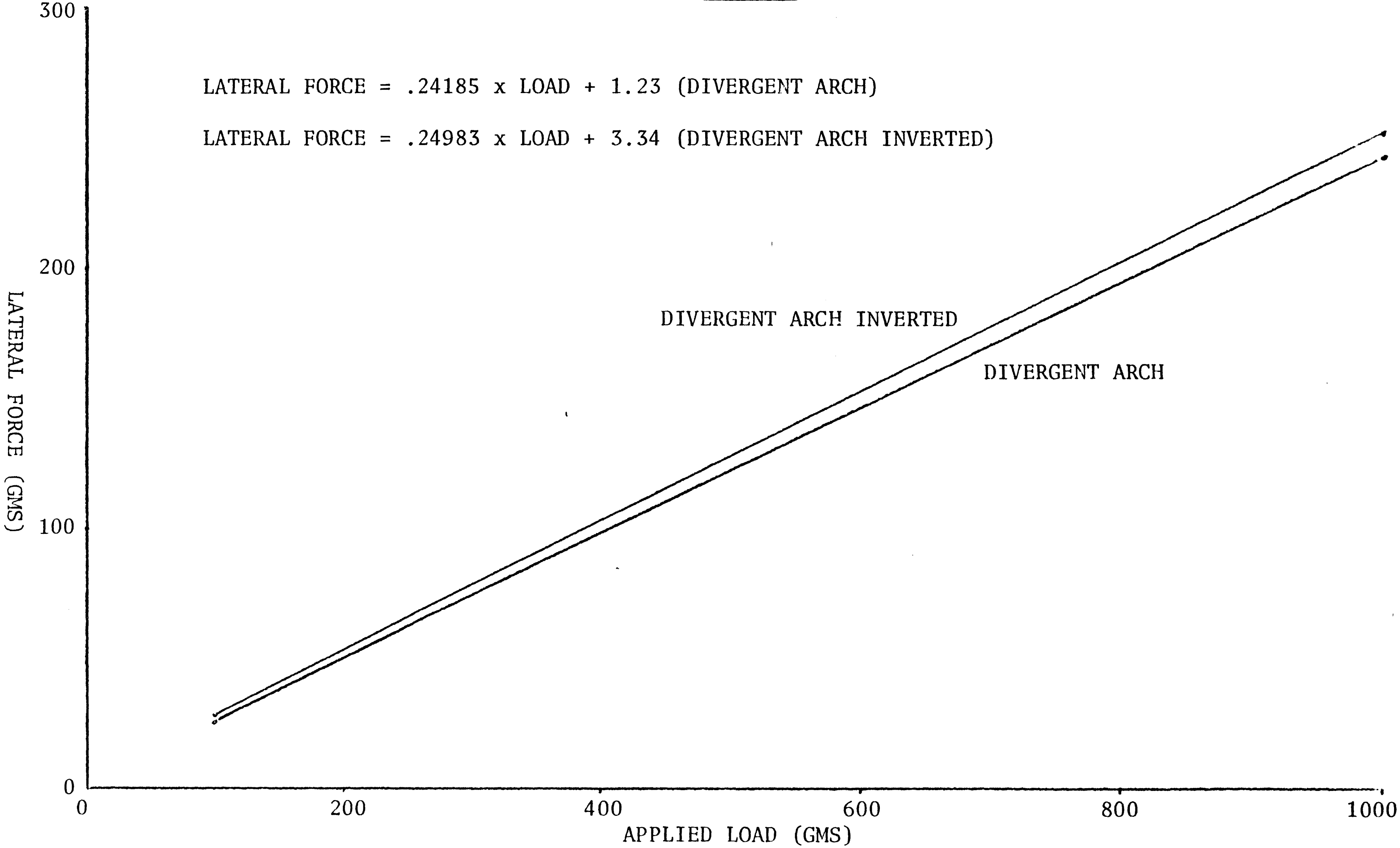
SIMPLE MODEL - RIGID ATTACHMENT - EXPERIMENTAL

PLOT I



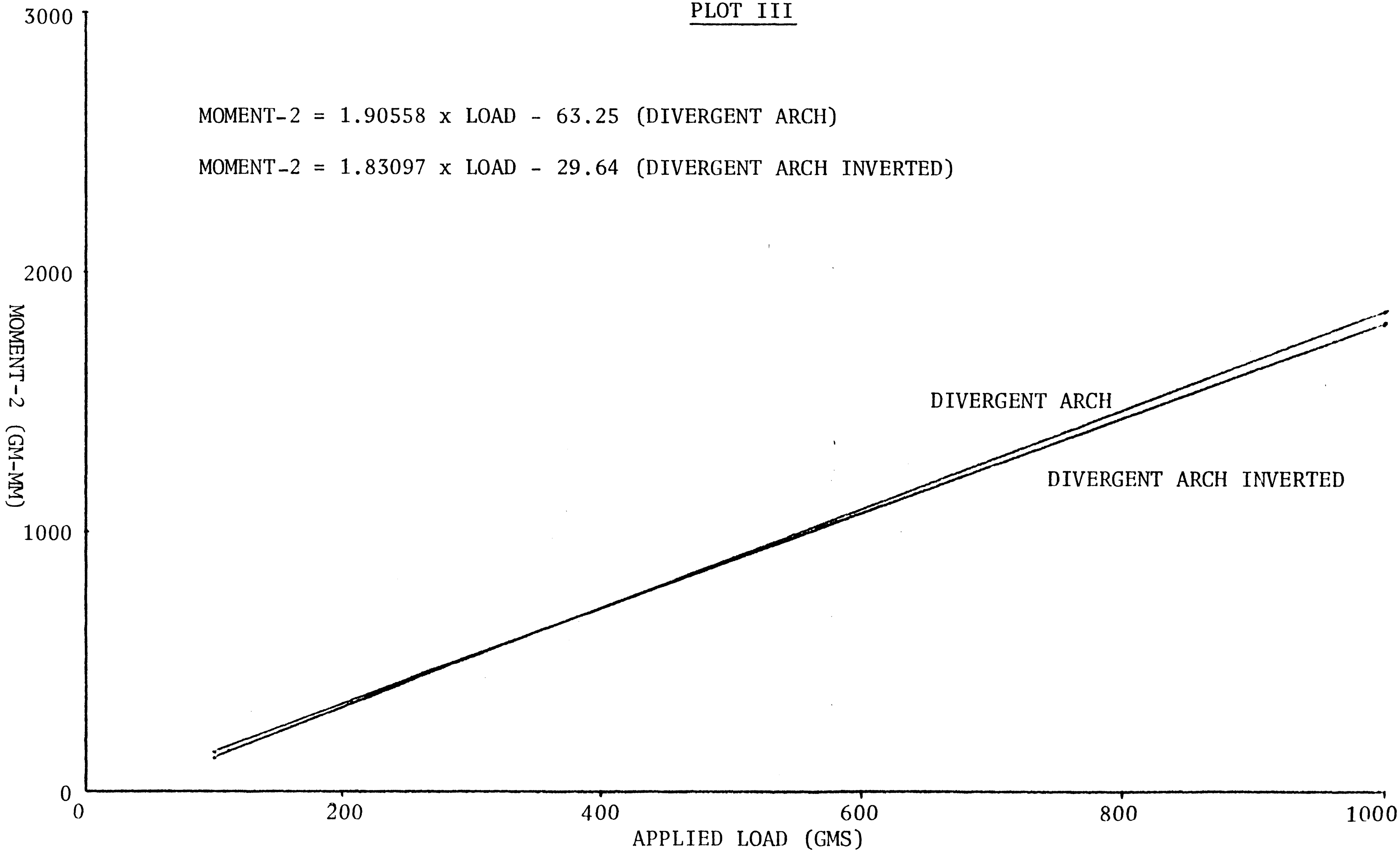
SIMPLE MODEL - RIGID ATTACHMENT - EXPERIMENTAL

PLOT II



SIMPLE MODEL - RIGID ATTACHMENT - EXPERIMENTAL

PLOT III



SIMPLE MODEL - RIGID ATTACHMENT - EXPERIMENTAL

PLOT IV

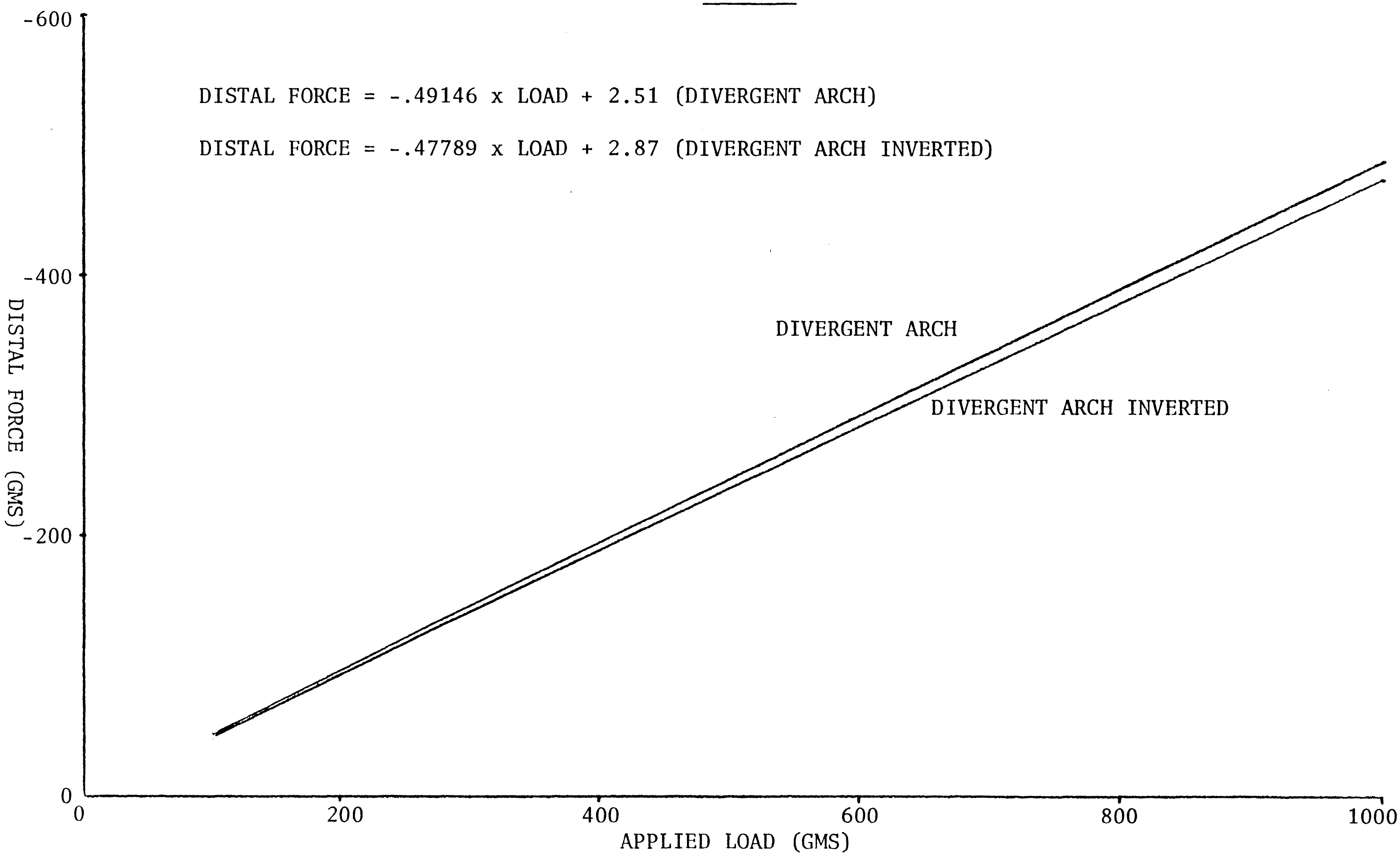


TABLE II
REGRESSION LINE COMPARISON OF DIVERGENT ARCHES-
ORIGINAL AND INVERTED ORIENTATIONS

COMPARISON	DEGREES OF FREEDOM	MOMENT-1	LATERAL FORCE	MOMENT-2	DISTAL FORCE
ELEVATIONS	1,88	.046 NS	.185 NS	.004 NS	.068 NS
SLOPES	1,86	1.074 NS	28.308 SD	13.736 SD	28.374 SD

SD = Significant Difference
NS = No Significant Difference

TABLE III
REGRESSION COEFFICIENTS (SLOPES),
INTERCEPTS, & CORRELATION COEFFICIENTS
FOR POOLED DIVERGENT ARCH DATA TESTED IN THE RIGID ATTACHMENT

ATTACHMENT	MODEL	STATISTIC	MOMENT-1	LATERAL FORCE	MOMENT-2	DISTAL FORCE
RIGID	DIVERGENT ARCH (POOLED)	INTERCEPT:	101.92	2.17	-48.32	2.67
		REGRESSION COEFFICIENT:	-1.7233	.2454	1.8724	-.48542
		CORRELATION COEFFICIENT:	-.98889	.99843	.99856	-.99922

APPENDIX XI

TABLE I

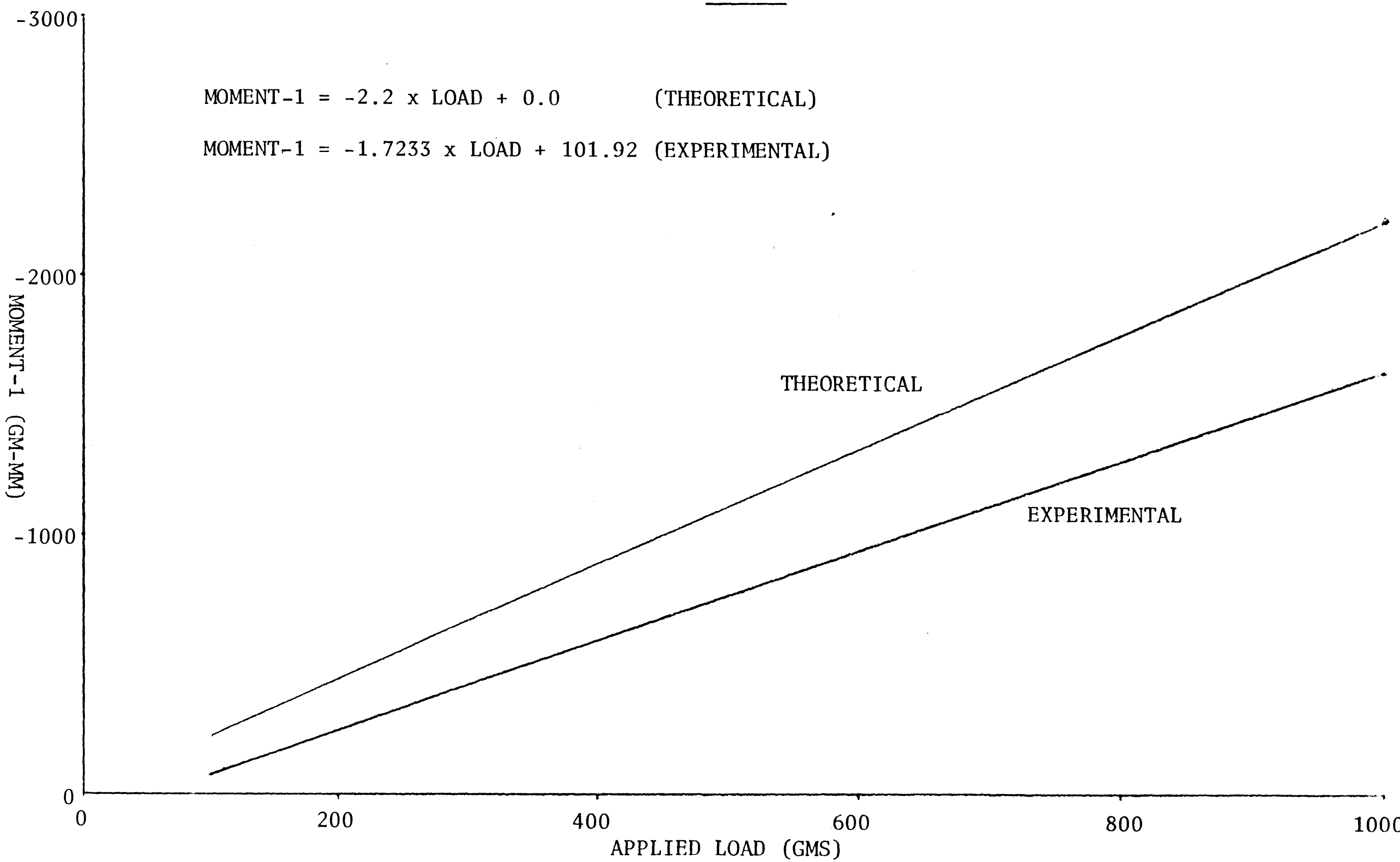
DIVERGENT ARCH - RIGID ATTACHMENT

THEORETICAL FORCE SYSTEM VALUES

LOAD (gms)	MOMENT-1 (gm-mm)	LATERAL FORCE (gms)	MOMENT-2 (gm-mm)	DISTAL FORCE (gms)
100	-220.0	28.0	240.0	-50.0
200	-440.0	56.0	480.0	-100.0
300	-660.0	84.0	720.0	-150.0
400	-880.0	112.0	960.0	-200.0
500	-1100.0	140.0	1200.0	-250.0
600	-1320.0	168.0	1440.0	-300.0
700	-1540.0	196.0	1680.0	-350.0
800	-1760.0	224.0	1920.0	-400.0
900	-1980.0	252.0	2160.0	-450.0
1000	-2200.0	280.0	2400.0	-500.0

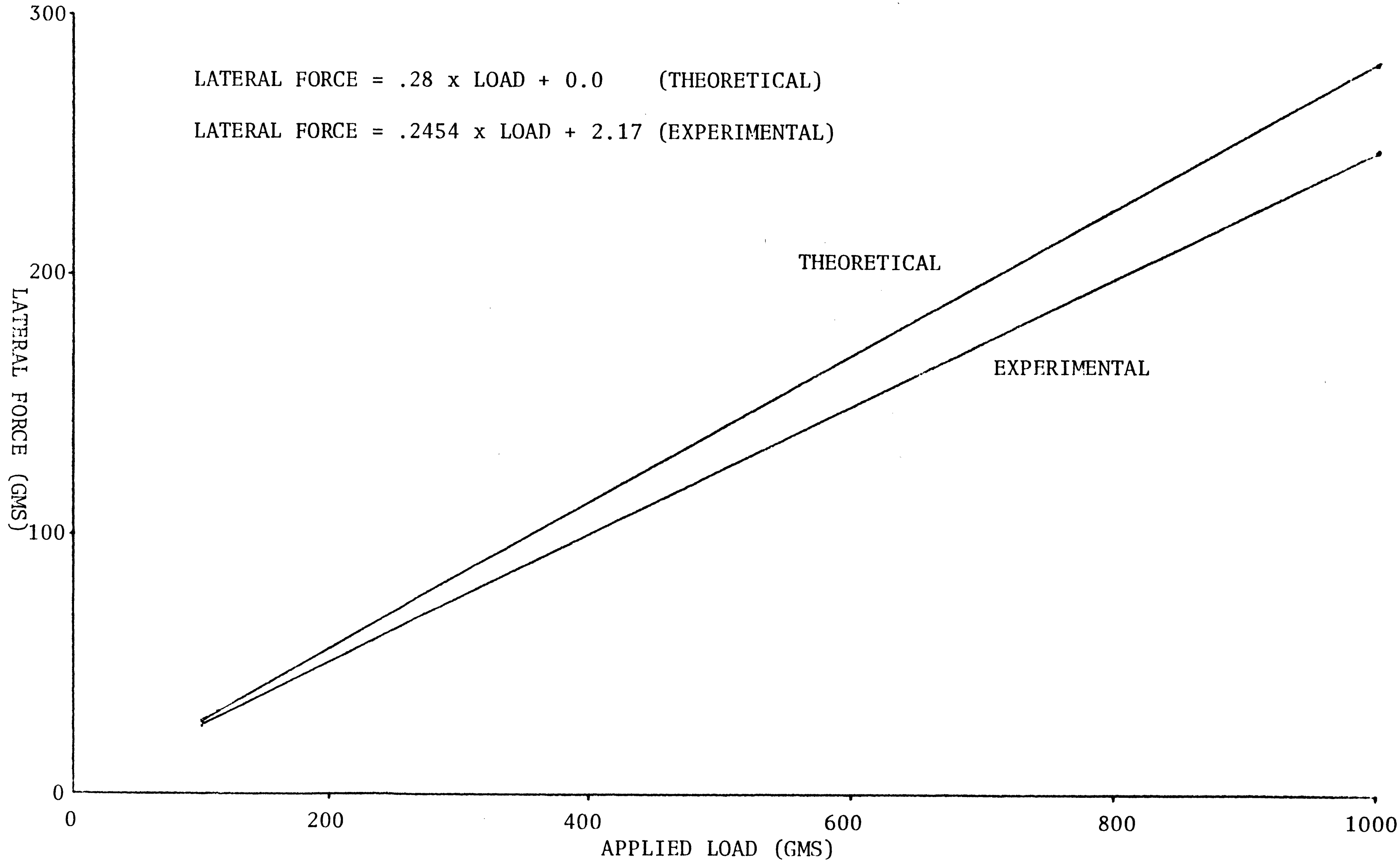
SIMPLE MODEL - RIGID ATTACHMENT - THEORETICAL vs EXPERIMENTAL

PLOT I



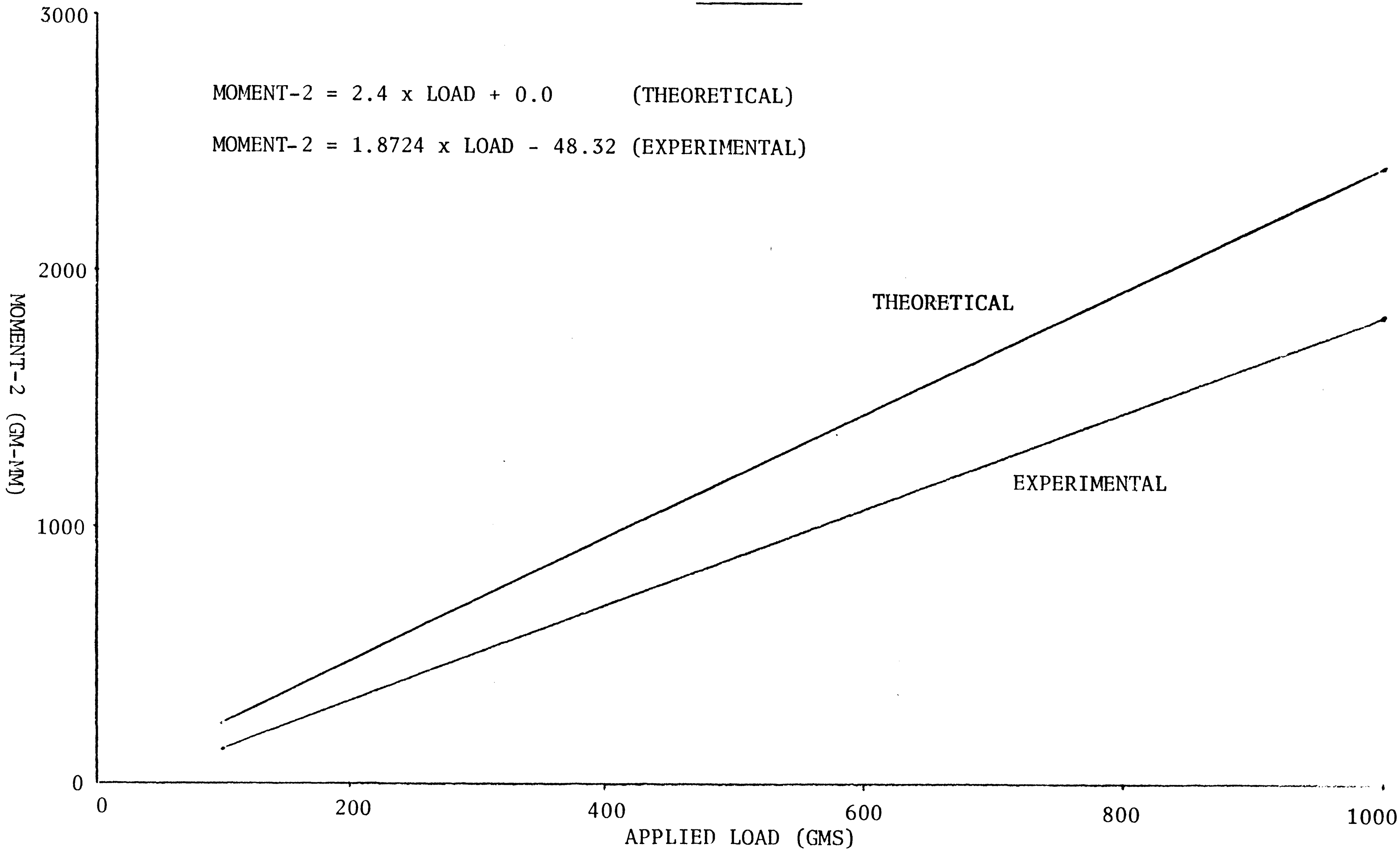
SIMPLE MODEL - RIGID ATTACHMENT - THEORETICAL vs EXPERIMENTAL

PLOT II



SIMPLE MODEL - RIGID ATTACHMENT - THEORETICAL vs EXPERIMENTAL

PLOT III



SIMPLE MODEL - RIGID ATTACHMENT - THEORETICAL vs EXPERIMENTAL

PLOT IV